OFDM-based Interference Alignment in Single-Antenna Cellular Wireless Networks

Huacheng Zeng, *Member, IEEE,* Yi Shi, *Senior Member, IEEE,* Y. Thomas Hou, *Fellow, IEEE,* Wenjing Lou, *Fellow, IEEE,* Xu Yuan, *Member, IEEE,* Rongbo Zhu, *Member, IEEE,* Jiannong Cao, *Fellow, IEEE*

***Abstract*—Interference alignment (IA) is widely regarded as a promising interference management technique in wireless networks. Despite its rapid advances in cellular networks, most results of IA are limited to information-theoretic exploration or physical-layer signal design. Little progress has been made so far to advance IA in cellular networks from a networking perspective. In this paper, we aim to fill this gap by studying IA in large-scale cellular networks. For the uplink, we propose an OFDM-based IA scheme and prove its feasibility at the physical layer by showing that all data streams in the IA scheme can be transported free of interference. Based on the IA scheme, we develop a cross-layer IA optimization framework that can fully translate the benefits of IA to throughput gain in cellular networks. Further, we show that the IA optimization problem in the downlink can be solved in the exactly same way as that in the uplink. Simulation results show that our OFDM-based IA scheme can significantly increase the user throughput and the throughput gain increases with user density in the network.**

***Index Terms*—Interference alignment, cellular networks, cross-layer optimization, throughput maximization**

I. Introduction

Interference alignment (IA) is a promising interference management technique in wireless networks as it may yield much higher throughput than we thought before. The basic idea of IA is to jointly construct signals at the transmitters with the aim of squeezing interfering signals into a reduced-dimensional subspace at each receiver, thereby leaving larger subspace for the reception of desired signals. It was shown by Camade and Jafar in [1] that IA makes it possible for the K-user interference channel to achieve $K/2$ degrees of freedom (DoFs), indicating that the aggregate DoFs of the interference channel increase linearly with the number of users. Given its huge potential, IA has gained tremendous momentum in the research community and been applied to a variety of networks (see, e.g., [2], [3], [4], [5]).

Along with its success in theory, IA in cellular networks (and WLAN) has attracted significant attention due to its industrial potentials. Research efforts have produced a flourish line of results that deepen our understanding of IA in cellular networks. For example, Suh et al. in [6] and [7] showed that the use of IA can completely eliminate inter-cell interference if the number of users in each cell is sufficiently large. Morales-Cespedes et al. in [8] developed an IA scheme using reconfigurable antennas to remove interference for partially connected cellular networks. Results of IA in cellular networks also include spatial IA design (see, e.g., [9], [10], [11]), blind IA design (see, e.g., [12], [13]), and channel state information (CSI) analysis (see, e.g., [14], [15]).

Although there is a large body of work on IA in cellular networks, most of them are limited to information-theoretic exploration or physical-layer signal design. It remains open how to develop an IA scheme that can be incorporated with upper-layer user scheduling algorithm to maximize network-level throughput. This stagnation underscores the technical challenges in the exploration of cross-layer IA design from a networking perspective, which we describe as follows. First, developing an IA scheme for a generic cellular network with an arbitrary number of base stations (BS) and users is not a trivial problem, as it requires complex signal design at transmitters and onerous signal detection at receivers. Second, in a large-scale network environment, IA design is coupled with upper-layer user scheduling. An isolated design of IA at the physical layer is prone to yield an inferior performance, and thus cannot fully harvest the benefits of IA for throughput maximization. Therefore, a cross-layer IA scheme is needed. However, developing a cross-layer IA scheme together with user scheduling can easily become intractable and is a challenging task.

In this paper, we study OFDM-based IA in large-scale cellular networks from a networking perspective. We consider a network that consists of a set of grid-deployed BSs and a set of randomly distributed users. Each BS has a fixed service area and provides service to the users within its service area. A user may fall into the service areas of multiple BSs and will choose one of them as its service provider. We assume the transmission is based on OFDM modulation and the set of available subcarriers in OFDM is given. For such a network, similar to the IA scheme in [6], [7], we study IA in the frequency domain by projecting the weighted transmit signals onto OFDM subcarriers. Doing so allows us to develop an IA scheme that can be easily applied to a network with heterogeneous antenna configurations. Given that uplink and downlink are independent in both TDD and FDD networks, we consider them separately. Our objective is to develop an OFDM-based IA scheme that can be jointly optimized with user scheduling to maximize the uplink/downlink user throughput in cellular networks. Although our study of IA is done in the frequency domain, the proposed IA scheme is a general framework which can also be used in the temporal and spatial domains. The contributions of this paper are summarized as follows:

- For the uplink, we develop an OFDM-based IA scheme

H. Zeng, Y. T. Hou, W. Lou, X. Yuan, and R. Zhu are with Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA. J. Cao is with The Hong Kong Polytechnic University, China.

- for the data transmissions from users to their serving BSs. Specifically, at each user, we propose an approach to determine which subset of its interfering streams should be selected for alignment; at each BS, we propose a procedure to align interfering streams so that the desired data streams can be decoded free of interference. For the proposed IA scheme, we develop a set of IA constraints for each user and BS, and show that if the IA constraints are satisfied, the IA scheme is always feasible at the physical layer.
- Based on the OFDM-based IA scheme, we develop a cross-layer IA optimization framework to maximize the user throughput for the uplink of cellular networks. To reduce the complexity of the optimization framework, we eliminate its nonlinear constraints through reformulation without compromising its optimality. The resulting optimization framework is in a form that can be easily handled by commercial off-the-shelf (COTS) optimization solvers.
- We show the uplink-downlink duality of the IA scheme. Specifically, we show that the uplink IA scheme can be applied to the downlink by simply switching the roles of user and BS. Further, the downlink IA optimization problem has the same formulation as the uplink and therefore can be solved in the exactly same way.
- We evaluate the throughput performance of our IA scheme via simulation. We compare it against two other schemes: "no-IA" scheme and "crude-IA" scheme. Simulation results show that our IA scheme has a significant throughput gain over no-IA and crude-IA schemes. Further, the gain of our IA scheme increases with user density of the network.

The remainder of this paper is organized as follows. Section II presents related work. Section III offers a primer of our OFDM-based IA scheme. In Section IV, we develop an OFDM-based IA scheme and prove its feasibility. In Section V, we develop an IA optimization framework to maximize network throughput. In Section VI, we establish the uplink-downlink duality of our IA scheme. Section VII presents numerical results to show the efficacy of the IA scheme and Section VIII concludes this paper.

II. Related Work

The idea of IA firstly appeared in [16] and the terminology of IA was created by Jafar and Shamai in a seminar paper for the two-user X channel [17]. Since its emergence, the idea has gained tremendous momentum in both industry and academia (see, e.g., [1], [18], [19], [5], [20], [21], [22], [23], [24]). Since there is an overwhelming large amount of work on IA, we cannot survey all the IA papers and therefore focus our literature survey on IA in cellular networks.

In [6] and [7], Suh et al. proposed a frequency-domain IA scheme, called subspace interference alignment, for both uplink and downlink of cellular networks. They further showed that their IA scheme can achieve $K/(\sqrt[G-1]{K}+1)^{G-1}$ DoFs for each cell, where G is the number of cells and K is the number of users in a cell. If the number of users per cell is large enough, each cell can achieve one DoF. This result is significant, as it indicates that interference may not be the dominant factor in cellular networks. Despite its significance, the IA scheme in [6] and [7] cannot be used in practical networks due to its underlying assumptions, including (i) only one data stream per user, (ii) identical user number for each BS, (iii) restricted relationship between subcarrier and user numbers, and (iv) a single interference collision domain. In contrast, the OFDM-based IA scheme in this paper does not rely on these assumptions, thereby making a concrete step towards its practical applications in cellular networks.

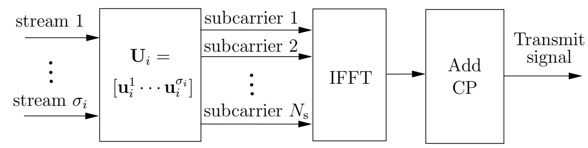


Fig. 1: Schematic diagram of IA at user i.

In addition to its advances in the frequency domain, IA was also studied in the spatial domain (using multiple antennas) for cellular networks [9], [10], [11], [25], [26], [15], [27]. In [9], Zhuang et al. investigated the feasibility of IA in MIMO cellular networks and proposed a max-SINR algorithm to design IA solutions. In [10], Shin et al. proposed an IA scheme to design transmit and receive beamforming vectors for a two-cell MIMO network and showed that their IA algorithm can achieve the optimal DoF. In [11], Ntranos et al. studied spatial-domain IA in cellular MIMO networks. They showed that their IA scheme can achieve 1/2 DoFs per antenna in the uplink of a three-sector cellular network with one active user per sector when both the user and the sector have M antennas.

Another research line of IA in cellular networks is focused on addressing its CSI problem. In [8], Morales-Cespedes et al. studied blind IA for partially-connected cellular networks and developed a blind IA scheme based on reconfigurable antenna to remove intra-cell and inter-cell interference. In [13], Wang et al. developed a blind IA scheme for the downlink of cellular clustered networks with reconfigurable antennas. In [12], Jose et al. studies the combination of IA and opportunistic scheduling to facilitate alignment in the cellular downlink while not requiring CSI at transmitters. In [14], Rao et al. first quantified CSI feedback for IA in MIMO cellular networks, and then derived closed-form tradeoff between the CSI feedback and IA performance. In [15], Tresch et al. studied the sum mutual information achieved by IA in cellular networks and derived its upper and lower performance bounds in the scenarios with imperfect channel knowledge.

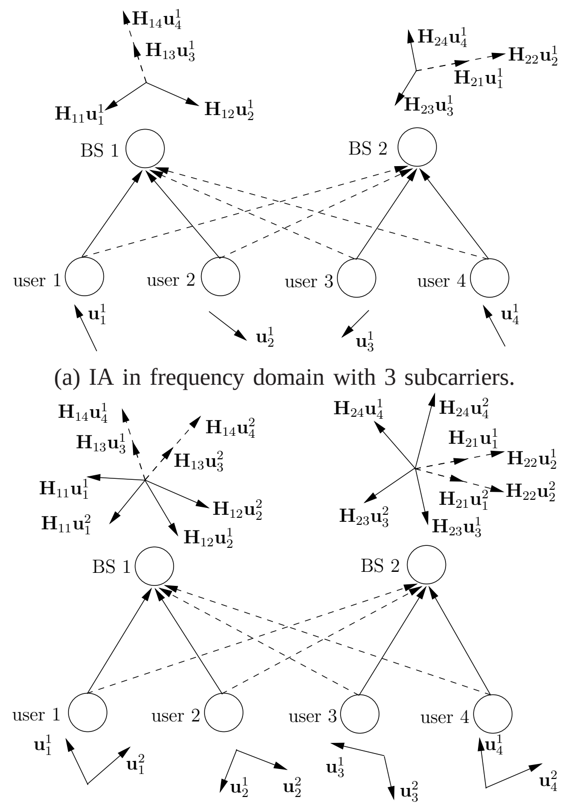
While there is a large amount of IA results in cellular networks, most of them were focused on information-theoretic exploration or physical-layer signal design. Little progress has been made to advance our understanding of IA from a networking perspective. This paper fills this gap by developing an IA scheme which can be jointly optimized with upper-layer user scheduling to maximize the throughput of cellular networks.

TABLE I: Notation.

Symbol	Definition
$\mathcal{A}_{ij}$	The subset of interfering streams at user i that are aligned at unintended BS j, $\mathcal{A}_{ij} \subseteq \mathcal{S}_i$, $\lvert\mathcal{A}_{ij}\rvert = \alpha_{ij}$
$\mathcal{B}_i$	The subset of interfering streams at user i that are not aligned at any BSs, $\mathcal{B}_i = \mathcal{S}_i \backslash (\cup_{j \in \mathcal{I}_i^{\mathrm{bs}}} \mathcal{A}_{ij})$, $\lvert\mathcal{B}_i\rvert = \beta_i$
$\mathcal{C}_i^{\mathrm{bs}}$	The set of BSs within the transmission range of user i
$\mathcal{C}_j^{\mathrm{usr}}$	The set of users that may choose BS j as service provider
$\mathcal{E}^{A_{ij}}$	The subset of precoding vectors that correspond to the interfering streams in $\mathcal{A}_{ij}$
$\mathcal{E}^{B_i}$	The subset of precoding vectors that correspond to the interfering streams in $\mathcal{B}_i$
$\mathcal{E}^{S_i}$	The set of precoding vectors that correspond to the streams in $\mathcal{S}_i$
$\mathbf{H}_{ji}$	Channel matrix from user i to BS j, $\mathbf{H}_{ji} \in \mathbb{C}^{K \times K}$
$\mathcal{I}_i^{\mathrm{bs}}$	The set of BSs that are interfered by user i
$\mathcal{I}_j^{\mathrm{usr}}$	The set of users that are interfering with BS j
K	The number of subcarriers in the network, $K = \lvert\mathcal{K}\rvert$
$\mathcal{K}$	The set of subcarriers in the network
M	The number of BSs in the network, $M = \lvert\mathcal{M}\rvert$
$\mathcal{M}$	The set of BSs in the network
N	The number of users in the network, $N = \lvert\mathcal{N}\rvert$
$\mathcal{N}$	The set of users in the network
$\mathcal{O}_i^{\mathrm{bs}}$	The set of BSs within the interference range of user i (but out of its transmission range)
$\mathcal{O}_j^{\mathrm{usr}}$	The set of users that interfere with BS j but out of BS j's service area
$\mathcal{Q}_j^{\mathrm{T}}$	The set of directions for desired data streams at BS j
$\mathcal{Q}_j^{\mathrm{I}}$	The set of directions for interfering streams at BS j
$\mathcal{Q}_j^{\mathrm{I,Eff}}$	The set of directions for "effective" interfering streams at BS j
$\mathcal{Q}_j^{\mathrm{I,Ali}}$	The set of directions for aligned interfering streams at BS j
$\mathcal{Q}_j^{\mathrm{I,Def}}$	The set of predefined directions for interference at BS j
s_i^k	The kth stream at user i, $1 \leq k \leq \sigma_i$
$\mathcal{S}_i$	The set of streams at user i, $\lvert\mathcal{S}_i\rvert = \sigma_i$
$\mathcal{T}_j^{\mathrm{usr}}$	The set of users that choose BS j as service provider
$\mathbf{u}_i^k$	The precoding vector for stream s_i^k over the subcarriers, $\mathbf{u}_i^k \in \mathbb{C}^{K \times 1}$
$\mathbf{v}_j^l$	The decoding vector for stream s_i^k over the subcarriers, $\mathbf{v}_j^l \in \mathbb{C}^{K \times 1}$
$r_{\min}$	The minimum data rate among all users
x_{ij}	A binary variable to indicate whether user i chooses BS j as its service provider
y_{ij}	An opposite binary variable of variable x_{ij}, $x_{ij} + y_{ij} = 1$
α_{ij}	The cardinality of $\mathcal{A}_{ij}$, $\alpha_{ij} = \lvert\mathcal{A}_{ij}\rvert$
β_i	The cardinality of $\mathcal{B}_i$, $\beta_i = \lvert\mathcal{B}_i\rvert$
σ_i	The number of streams at user i, $\sigma_i = \lvert\mathcal{S}_i\rvert$
λ_{ij}	Linearization variable, $\lambda_{ij} = \alpha_{ij} \cdot y_{ij}$
μ_{ij}	Linearization variable, $\mu_{ij} = \beta_i \cdot y_{ij}$

III. OFDM-based IA in Cellular Networks: A Primer

IA is a promising interference management technique in wireless networks. Its basic idea is to jointly design signals at transmitters using linear precoding techniques, with the aim of projecting interfering signals into a reduced-dimensional subspace and keeping the desired signals resolvable at each receiver. Generally speaking, IA can be done in three domains: spatial (MIMO), spectral (OFDM), and temporal (time slots). In this paper, we consider OFDM-based IA in the frequency domain in cellular networks. We assume that the transmission is using OFDM modulation and the set of subcarriers available for IA is given. At each transmitter, as shown in Fig. 1, IA is achieved by projecting its outgoing data streams onto the subcarriers using linear precoding technique. As such, the core of IA is a construction of precoding vectors for the outgoing data streams at the transmitters. Suppose that there are K (e.g., 64) subcarriers available in the network. Then the precoding vector for each data stream is a $K \times 1$ complex vector. At each transmitter, we aim to design a precoding vector for each of its data streams so that its transmitted signals overlap as much as possible at its unintended receiver(s) while remaining resolvable at its intended receiver(s). Table I lists the notation that we use in the paper.



(a) IA in frequency domain with 3 subcarriers.

(b) IA in frequency domain with 6 subcarriers.

Fig. 2: An example of IA in the frequency domain.

An Example. Consider a small network with 2 BSs and 4 users as shown in Fig. 2, where a solid arrow line represents a directed link and a dashed arrow line represents a directed interference. For both BSs and users, each of them has a single antenna. We assume that CSI is available at both BSs and users. To show the benefits of IA, let's start with a simple example by assuming 3 subcarriers available for data transmission (i.e., $K = 3$). Note that we take $K = 3$ only for ease of illustration and we will consider a larger value of K later. With $K = 3$, we show that by using IA, a total of 4 data streams can be transmitted from the users to their respective BSs, with 1 data stream from each user.

To show this, we first introduce the notation. For vectors $\mathbf{a}$ and $\mathbf{b}$, denote $\mathbf{a} := \mathbf{b}$ if there exists a nonzero complex number c such that $\mathbf{a} = c\mathbf{b}$, i.e., $\mathbf{a}$ and $\mathbf{b}$ are in the same direction. Denote $\{\mathbf{u}_{\mathrm{b}}^1 \cdots \mathbf{u}_{\mathrm{b}}^K\}$ as a set of linearly independent basis vectors with dimension $K \times 1$ and nonzero entries. The independence requirement ensures that the data streams from the same user remain resolvable at the BS; and the nonzero requirement ensures that all subcarriers are fully used. Denote

$\mathbf{H}_{ji}$ as the frequency-domain channel matrix between user i to BS j. Due to the orthogonality of the subcarriers, $\mathbf{H}_{ji}$ is a diagonal square matrix. Denote $\mathbf{u}_i^k$ as the precoding vector for the kth outgoing stream at user i. We construct the precoding vectors at user 1 and user 2 as follows: let $\mathbf{u}_1^1 := \mathbf{u}_{\mathrm{b}}^1$ and let $\mathbf{u}_2^1 := \mathbf{H}_{22}^{-1}\mathbf{H}_{21}\mathbf{u}_1^1$. As a result, at BS 2, the interfering stream from user 1 is aligned to the interfering stream from user 2, as shown in Fig. 2(a). Likewise, we construct the precoding vectors at user 3 and user 4 as follows: let $\mathbf{u}_3^1 := \mathbf{u}_{\mathrm{b}}^2$ and let $\mathbf{u}_4^1 := \mathbf{H}_{24}^{-1}\mathbf{H}_{23}\mathbf{u}_3^1$. Then at BS 1, the interfering stream from user 3 is aligned to the interfering stream from user 4, as shown in Fig. 2(a). By using the above precoding vectors at the 4 users, the received data and interfering streams at each BS are on 3 different directions. Therefore, 3 subcarriers are sufficient to support 4 data streams. However, if IA is not used, 3 subcarriers can support only 3 data streams from the four users (with any combinations), since putting more than one data stream on a subcarrier will inevitably cause interference on that subcarrier.

When the network has 6 subcarriers (i.e., $K = 6$), we show that by using IA, 8 data streams can be transmitted from the users to their BSs, with 2 data stream at each user. We construct the precoding vectors at user 1 and user 2 as follows: let $[\mathbf{u}_1^1\ \mathbf{u}_1^2] := [\mathbf{u}_{\mathrm{b}}^1\ \mathbf{u}_{\mathrm{b}}^2]$ and let $[\mathbf{u}_2^1\ \mathbf{u}_2^2] := \mathbf{H}_{22}^{-1}\mathbf{H}_{21}[\mathbf{u}_1^1\ \mathbf{u}_1^2]$. As a result, at BS 2, the two interfering streams from user 1 are aligned to the two interfering streams from user 2, as shown in Fig. 2(b). Likewise, we construct the precoding vectors at user 3 and user 4 as follows: let $[\mathbf{u}_3^1\ \mathbf{u}_3^2] := [\mathbf{u}_{\mathrm{b}}^3\ \mathbf{u}_{\mathrm{b}}^4]$ and let $[\mathbf{u}_4^1\ \mathbf{u}_4^2] := \mathbf{H}_{24}^{-1}\mathbf{H}_{23}[\mathbf{u}_3^1\ \mathbf{u}_3^2]$. As a result, at BS 1, the two interfering streams from users 3 are aligned to the two interfering streams from user 4, as shown in Fig. 2(b). By using those precoding vectors at the 4 users, the received data and interfering streams at each BS are on 6 directions. Therefore, 6 subcarriers are sufficient to support 8 data streams. However, if IA is not used, 6 subcarriers can support only 6 data streams from the four users (with any combinations), since putting more than one data stream on a subcarrier will lead to interference. Following the same token, when the network has 256 subcarriers, it can transport 340 data streams using IA, in contrast of 256 data streams in the case without IA. □

It is easy to see that the gain of IA for this network is $1/3$. It is worth pointing out that the gain of IA becomes more significant as the number of users increases. In the two-BS cellular network where each BS has n users, the interferences at each BS can be aligned to the same direction. The spectrum efficiency at each BS is $n/(n+1)$ and the total spectrum efficiency at the two BSs is $2n/(n+1)$. Since the network without IA is 1, the gain of IA is $(n-1)/(n+1)$.

IV. An OFDM-based IA Scheme and Its Feasibility

In this section, we develop an OFDM-based IA scheme for the uplink communication in a single-antenna cellular wireless network. The IA scheme includes IA constraints at each user and BS, as well as how to construct the precoding/decoding vectors for each stream. In what follows, we first present the OFDM-based IA scheme and then prove its feasibility at the physical layer.

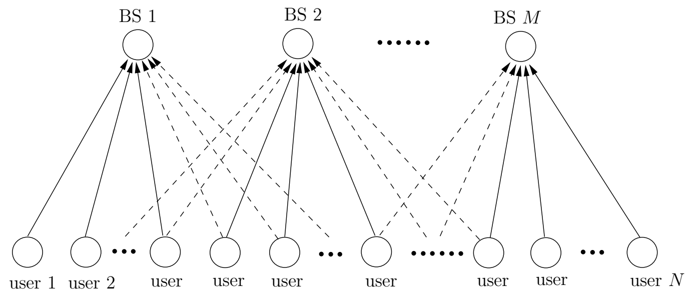


Fig. 3: The uplink transmission in a cellular network.

A. An OFDM-based IA Scheme

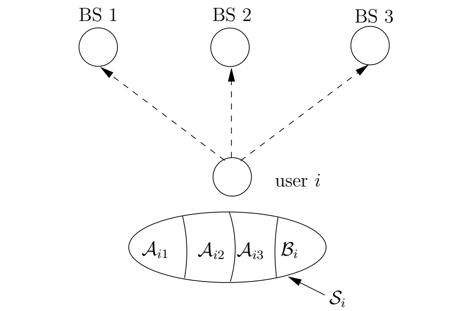
Consider a cellular network shown in Fig. 3, where each node (BS or user) has a single antenna. Denote $\mathcal{N}$ as the set of users in the network with $N = |\mathcal{N}|$. Denote $\mathcal{M}$ as the set of BSs in the network $M = |\mathcal{M}|$. Denote $\mathcal{T}_j^{\mathrm{usr}}$ as the set of users who choose BS j as their service provider. Denote $\mathcal{I}_j^{\mathrm{usr}}$ as the set of users that interfere with BS j, i.e., BS j is within the interference range of these users and BS j is not the service provider of these users. Denote $\mathcal{I}_i^{\mathrm{bs}}$ as the set of BSs that are interfered with by user i, i.e., these BSs are within the interference range of user i but are not chosen by user i as its service provider. In our study, we use Rayleight fading as the channel fast fading model. For the channel realizations, we assume that the CSI is available at both BSs and users.

Consider user i that interferes with BS j, i.e., $j \in \mathcal{I}_i^{\mathrm{bs}}$. Denote $\mathcal{S}_i = \{s_i^k : 1 \le k \le \sigma_i\}$ as the set of streams from user i, where s_i^k is the kth stream and σ_i is the number of streams at user i (i.e., $\sigma_i = |\mathcal{S}_i|$). Then each stream in $\mathcal{S}_i$ is an interfering stream for BS j. At BS j, we wish to align as many interfering streams as possible to some predefined interference directions.

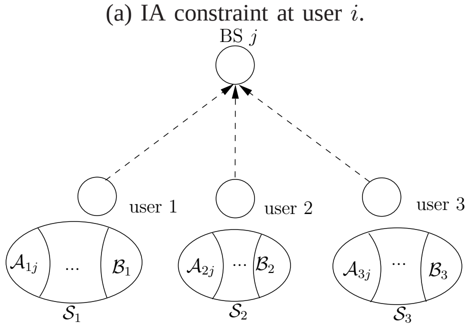
Among the interfering streams in $\mathcal{S}_i$, denote $\mathcal{A}_{ij}$ as the subset of interfering streams that can be aligned to some predefined interference directions at BS j. Denote α_{ij} as the cardinality of $\mathcal{A}_{ij}$, i.e., $\alpha_{ij} = |\mathcal{A}_{ij}|$. Then, at BS j, the number of directions occupied by the interfering streams is reduced from σ_i to $\sigma_i - \alpha_{ij}$, resulting in a saving of α_{ij} directions at BS j. Among the streams in $\mathcal{S}_i$, there may be a subset $\mathcal{B}_i$ of streams that are not aligned to any predefined interference direction at the BSs in $\mathcal{I}_i^{\mathrm{bs}}$. Denote β_i as the cardinality of $\mathcal{B}_i$, i.e., $\beta_i = |\mathcal{B}_i|$. Thus we have

$$\mathcal{B}_i = \mathcal{S}_i \backslash (\cup_{j \in \mathcal{I}_i^{\mathrm{bs}}} \mathcal{A}_{ij}).$$

Constraints at User. Consider one of the σ_i outgoing data streams at user i as shown in Fig. 4(a). This outgoing data stream interferes with all the BSs in $\mathcal{I}_i^{\mathrm{bs}}$ (BS 1, 2, and 3 in the figure). We wish to align its produced interference at as many BSs as possible. Now the question is that the interference produced by this data stream can be aligned at how many BSs in $\mathcal{I}_i^{\mathrm{bs}}$. As we showed in the example in Section III, the construction of this data stream's precoding vector can guarantee its produced interference to be aligned at one BS. Note that in some extreme circumstances (e.g., when all the channels are exactly the same), the construction of this data stream's precoding vector can align its produced interference



(a) IA constraint at user i.



(b) IA constraint at BS j.

Fig. 4: An example of illustrating IA constraints at user i and BS $j \in \mathcal{I}_i^{\rm bs}$.

at multiple BSs. But in a general case, the construction of this data stream's precoding vector can guarantee the alignment of its interference at only one BS.

We now consider all the σ_i outgoing data streams at user i. Since each of them guarantees that one of its produced interfering streams can be successfully aligned at the corresponding BS. The construction of the σ_i outgoing data streams' precoding vectors guarantees that σ_i of their produced interfering streams can be successfully aligned at the corresponding BSs. To ensure the feasibility of IA, we impose constraint $\sum_{j\in\mathcal{I}_i^{\rm bs}} \alpha_{ij} \le \sigma_i$. Recall that β_i is a non-negative integer optimization variable. The constraint can be equivalently translated to

$$\beta_i + \sum_{j\in\mathcal{I}_i^{\rm bs}} \alpha_{ij} = \sigma_i, \quad \text{for } i \in \mathcal{N}. \tag{1}$$

Constraints at BS. At BS j (see Fig. 4(b) for example), we need to align the interfering streams in $\mathcal{A}_{ij}$ (for each $i \in \mathcal{I}_j^{\rm usr}$) to some predefined interference directions. To do so, we have the following two questions: (i) what should be the set of predefined interference directions at BS j; (ii) how to align the interfering streams in $\mathcal{A}_{ij}$ to the set of predefined interference directions.

There may be many possible solutions to the above two questions. Here, we show one solution for which we can offer a feasibility proof (see Section IV-B). In our solution, for the first question, we use $\cup_{i\in\mathcal{I}_j^{\rm usr}} \mathcal{B}_i$ as the set of predefined interference directions at BS j. That is, each interfering stream in $\mathcal{A}_{ij}$ will be aligned to an interfering stream in $\cup_{i\in\mathcal{I}_j^{\rm usr}} \mathcal{B}_i$. For the second question, we align each interfering stream in $\mathcal{A}_{ij}$, $i \in \mathcal{I}_j^{\rm usr}$, to a unique interference stream in $\cup_{k\in\mathcal{I}_j^{\rm usr}}^{k\neq i} \mathcal{B}_k$. That is, each interfering stream in $\mathcal{A}_{ij}$ is aligned uniquely in the interference subspace formed by the union of $\mathcal{B}_k$ over $k \in \mathcal{I}_j^{\rm usr}$ except its own $\mathcal{B}_i$. Here, "uniquely" means that any two interfering streams in $\mathcal{A}_{ij}$ will not be aligned to the same interfering stream in $\cup_{k\in\mathcal{I}_j^{\rm usr}}^{k\neq i} \mathcal{B}_k$. Based on our proposed solution to questions (i) and (ii), we have the following constraints at BS j:

$$\alpha_{ij} \le \sum_{k\in\mathcal{I}_j^{\rm usr}}^{k\neq i} \beta_k, \quad \text{for } i \in \mathcal{I}_j^{\rm usr}, j \in \mathcal{M}. \tag{2}$$

Dimension Constraints. At BS j, the total number of its desired data streams is $\sum_{i\in\mathcal{T}_j^{\rm usr}} \sigma_i$, while the number of its unaligned interfering streams is $\sum_{i\in\mathcal{I}_j^{\rm usr}} (\sigma_i - \alpha_{ij})$. Since the number of directions for desired data streams and unaligned interfering streams cannot exceed the number of available subcarriers, we have the following constraints at BS j:

$$\sum_{i\in\mathcal{T}_j^{\rm usr}} \sigma_i + \sum_{i\in\mathcal{I}_j^{\rm usr}} (\sigma_i - \alpha_{ij}) \le K, \quad \text{for } j \in \mathcal{M} \ . \tag{3}$$

So far we have derived three constraints for IA to characterize its capability (i.e., the number of data streams that can be sent by each user to its serving BS) without rigorous argument. In the next subsection, we show that as long as these three constraints are satisfied, there always exist precoding/decoding vectors so that σ_i data streams can be sent from user i to its serving BS free of interference, $i \in \mathcal{N}$.

B. Feasibility of the IA Scheme

Consider an IA scheme π with DoF vector $(\sigma_1, \sigma_2, \cdots, \sigma_N)$. For each stream s_i^k in π, denote $\mathbf{u}_i^k$ as its precoding vector at user i and $\mathbf{v}_j^l$ as its decoding vector at its intended BS j. We say IA scheme π is feasible if there exist precoding and decoding vectors so that user i can send σ_i data streams to its intended BS j free of interference, $i \in \mathcal{N}$. Then we have

Definition 1: An IA scheme π is feasible at the physical layer if there exist precoding and decoding vectors that meet

$$(\mathbf{v}_j^l)^T \mathbf{H}_{ji} \mathbf{u}_i^k = 1, \tag{4}$$

$$(\mathbf{v}_j^l)^T \mathbf{H}_{ji'} \mathbf{u}_{i'}^{k'} = 0, \quad i' \in \mathcal{T}_j^{\rm usr} \cup \mathcal{I}_j^{\rm usr}, 1 \le k' \le \sigma_{i'}, (i', k') \neq (i, k), \tag{5}$$

for $i \in \mathcal{N}$ and $1 \le k \le \sigma_i$.

Note that $\mathbf{H}_{ji}$, the frequency-domain channel matrix between user i and BS j, is a diagonal complex matrix with the kth diagonal entry being channel coefficient of the kth subcarrier. The following theorem is the main result of the rest of this subsection.

Theorem 1: For uplink IA scheme π, if its DoF vector $(\sigma_1, \sigma_2, \cdots, \sigma_N)$ satisfies (1), (2), and (3), then it is feasible at the physical layer.

Theorem 1 provides a sufficient condition to verify the feasibility of an IA scheme. Instead of constructing precoding and decoding vectors satisfying (4) and (5) in Definition 1, Theorem 1 allows to verify the feasibility of an IA scheme through simple calculation in (1), (2), and (3). The rest of section will be devoted to proving Theorem 1. Here is our road map. First, we construct precoding vectors for the data streams at each user. Second, we give two lemmas to characterize the dimensions of such precoding vectors. Finally, based on those two lemmas, we show that there always exists a decoding vector for each stream so that (4) and (5) in Definition 1 are satisfied.

Construction of Precoding Vectors. Denote $\mathcal{E}^{S_i} = \{\mathbf{u}_i^k : 1 \le k \le \sigma_i\}$ as the set of precoding vectors for the streams in $\mathcal{S}_i$ at user i. Among the precoding vectors in $\mathcal{E}^{S_i}$, denote $\mathcal{E}^{A_{ij}}$ as the subset of precoding vectors that correspond to the interfering streams in $\mathcal{A}_{ij}$; denote $\mathcal{E}^{B_i}$ as the subset of precoding vectors that correspond to the interfering streams in $\mathcal{B}_i$. Since we define a unique precoding vector for each stream, we have

$$\begin{aligned}
&|\mathcal{E}^{A_{ij}}| = \alpha_{ij}, \quad \text{for } j \in \mathcal{M}, i \in \mathcal{I}_j^{\text{usr}};\\
&|\mathcal{E}^{B_i}| = \beta_i, \quad \text{for } i \in \mathcal{N};\\
&\mathcal{E}^{B_i} = \mathcal{E}^{S_i} \backslash (\cup_{j\in\mathcal{I}_i^{\text{bs}}} \mathcal{E}^{A_{ij}}), \quad \text{for } i \in \mathcal{N};\\
&\mathcal{E}^{A_{ij_1}} \cap \mathcal{E}^{A_{ij_2}} = \emptyset, \quad \text{for } i \in \mathcal{N}, j_1, j_2 \in \mathcal{I}_i^{\text{bs}}, j_1 \neq j_2.
\end{aligned}$$

We define $\mathcal{E}^A = \cup_{i\in\mathcal{N}, j\in\mathcal{I}_i^{\text{bs}}} \mathcal{E}^{A_{ij}}$ and $\mathcal{E}^B = \cup_{i\in\mathcal{N}} \mathcal{E}^{B_i}$. Then we have $\cup_{i\in\mathcal{N}} \mathcal{E}^{S_i} = \mathcal{E}^A \cup \mathcal{E}^B$. We first construct the precoding vectors in $\mathcal{E}^B$ and then construct the precoding vectors in $\mathcal{E}^A$.

Denote $\{\mathbf{u}_{\text{b}}^k : 1 \le k \le K\}$ as a set of *linear independent* complex vectors with dimension $K \times 1$ and *nonzero* entries. Then, for the precoding vectors in $\mathcal{E}^B$, we construct each of them as follows:

$$\mathbf{u}_i^k := \mathbf{u}_{\text{b}}^k \,. \tag{6}$$

Now we construct the precoding vectors in $\mathcal{E}^A$. Recall that in IA scheme π, each interfering stream in $\mathcal{A}_{ij}$ is aligned to an interfering stream in $\cup_{k\in\mathcal{I}_j^{\text{usr}}}^{k\neq i} \mathcal{B}_k$. Therefore, for each $\mathbf{u}_i^k \in \mathcal{E}^A$, we define

$$\mathbf{u}_i^k := \mathbf{H}_{ji}^{-1} \mathbf{H}_{ji'} \mathbf{u}_{i'}^{k'} \,. \tag{7}$$

where $\mathbf{u}_{i'}^{k'}$ is an precoding vector in $\mathcal{E}^B$ (i.e., $\mathbf{u}_{i'}^{k'} := \mathbf{u}_{\text{b}}^{k'}$) and $i' \neq i$.

Properties of Precoding Vectors. Denote $\dim(\mathcal{E}^{S_i})$ as the dimension of the subspace spanned by the vectors in set $\mathcal{E}^{S_i}$. Then we have the following lemma:

Lemma 1: At each user $i \in \mathcal{N}$, the constructed precoding vectors $\mathcal{E}^{S_i}$ are linearly independent, i.e., $\dim(\mathcal{E}^{S_i}) = |\mathcal{E}^{S_i}|$.

A proof of Lemma 1 is given in Appendix A.

At BS j, denote $\mathcal{Q}_j^{\text{T}}$ as the set of its directions for its desired data streams and $\mathcal{Q}_j^{\text{I}}$ as the set of directions for its interfering streams. Mathematically, we have

$$\begin{aligned}
\mathcal{Q}_j^{\text{T}} &= \cup_{i\in\mathcal{T}_j^{\text{usr}}} \{\mathbf{H}_{ji}\mathbf{u}_i^k : \mathbf{u}_i^k \in \mathcal{E}^{S_i}\},\\
\mathcal{Q}_j^{\text{I}} &= \cup_{i\in\mathcal{I}_j^{\text{usr}}} \{\mathbf{H}_{ji}\mathbf{u}_i^k : \mathbf{u}_i^k \in \mathcal{E}^{S_i}\}.
\end{aligned}$$

Then, we have the following lemma:

Lemma 2: At each BS $j \in \mathcal{M}$, each of its desired data stream occupies an independent direction, i.e.,

$$\dim(\mathcal{Q}_j^{\text{T}} \cup \mathcal{Q}_j^{\text{I}}) = \sum_{i\in\mathcal{T}_j^{\text{usr}}} \sigma_i + \dim(\mathcal{Q}_j^{\text{I}}), \ \textit{for } j \in \mathcal{M}. \tag{8}$$

A proof of Lemma 2 is given in Appendix B.

Existence of Decoding Vectors. So far we have constructed precoding vectors for the streams at each transmitter and showed two important properties of the constructed. For the decoding vectors at receivers, we have the following proposition:

Proposition 1: If the constructed precoding vectors satisfy (8), then there exists a decoding vector for each stream so that constraints (4) and (5) are satisfied.

A proof of Proposition 1 is given in Appendix C. This completes the proof of Theorem 1.

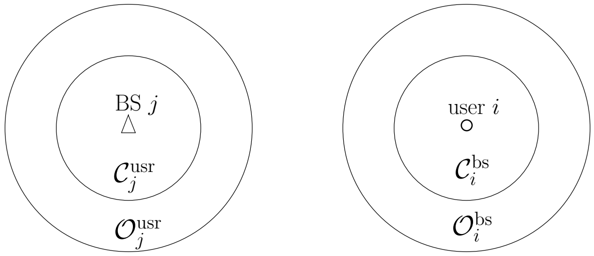
V. A Throughput Optimization Framework: Combining IA with User Scheduling

Problem Statement. Our goal is to exploit the benefits of OFDM-based IA to increase user throughput in cellular networks from a networking perspective. We consider a network that consists of a set of grid-deployed BSs and a set of randomly distributed users (see e.g., Fig. 7). Each BS has a fixed service area (a disk with radius of its transmission range) and it only provides service to the users within its service area. A user may fall into the service areas of multiple BSs and will choose one of them as its service provider. In this section, we focus on the IA optimization for the uplink. The downlink will be considered in the next section. In the uplink, a user is transmitter and it will interfere with the BSs within its interference range other than its service provider. We assume the transmission uses OFDM modulation and the set of subcarriers for IA is given. Within the set of subcarriers, we aim to jointly optimize IA and user scheduling so that the uplink user throughput can be maximized at network level.

Our Approach. To solve this problem, we develop a cross-layer IA optimization framework with the objective of maximizing user throughput. In the previous section we developed an IA scheme and showed that as long as IA constraints (1)–(3) are satisfied, user $i \in \mathcal{N}$ can send σ_i data streams to its serving BS free of interference. Constraints (1)–(3) define a feasible IA design space for a cellular network that is readily used for throughput optimization. However, constraints (1)–(3) were derived under the assumption that each user's serving BS is given a priori and therefore can only be applied to a network with static user access. In a practical network, as stated earlier, a user may be within the service area of multiple BSs and can choose any of them as its serving BS. This freedom (i.e., user scheduling at each BS) provides another dimension for IA optimization space to improve user throughput and should be incorporated into the IA design space. In what follows, we first model the user scheduling and then incorporate the user scheduling into our developed IA constraints. Finally, we formulate a user throughput optimization framework and eliminate its nonlinear constraints without loss of optimality.

A. Combining IA constraints with User Scheduling

As shown in Fig. 5(a), denote $\mathcal{C}_j^{\text{usr}}$ as the set of users within the service area of BS j; denote $\mathcal{O}_j^{\text{usr}}$ as the set of users that are outside the service area of BS j but can still interfere with BS j. As shown in Fig. 5(b), denote $\mathcal{C}_i^{\text{bs}}$ as the set of BSs that user i can choose as its service provider; denote $\mathcal{O}_i^{\text{bs}}$ as



(a) Sets for BS j. (b) Sets for user i.

Fig. 5: Sets illustration at BS j and user i.

the set of BSs whose service areas do not cover user i but are still inside the interference range of user i. To efficiently use IA and IC capabilities, channel quality (path loss and slow fading) should be taken into account when constructing these sets (i.e., $\mathcal{C}_j^{\text{usr}}$, $\mathcal{O}_j^{\text{usr}}$, $\mathcal{C}_i^{\text{bs}}$, and $\mathcal{O}_i^{\text{bs}}$).

Denote x_{ij} as a binary variable to indicate whether or not user i chooses BS $j \in \mathcal{C}_i^{\text{bs}}$ as its service provider. Specifically, $x_{ij} = 1$ if user i chooses BS j as its service provider and 0 otherwise. Since user i can choose only one BS as its service provider, we have

$$\sum_{j \in \mathcal{C}_i^{\text{bs}}} x_{ij} = 1, \quad i \in \mathcal{N}. \tag{9}$$

Denote y_{ij} as the complementary binary variable of x_{ij}. That is, $y_{ij} = 1$ if user i does *not* choose BS $j \in \mathcal{C}_i^{\text{bs}}$ as its service provider and 0 otherwise. Then we have the following constraints.

$$x_{ij} + y_{ij} = 1, \quad j \in \mathcal{C}_i^{\text{bs}}, i \in \mathcal{N}. \tag{10}$$

We now show that the above BS selection (user scheduling) variables can be incorporated into (1), (2), and (3) in our IA scheme. To incorporate BS selection variables in (1), we need to first clarify $\mathcal{I}_i^{\text{bs}}$, i.e., the set of BSs that are interfered with by user i. Based on the definitions of $\mathcal{O}_i^{\text{bs}}$, $\mathcal{C}_i^{\text{bs}}$, and y_{ij}, we have

$$\mathcal{I}_i^{\text{bs}} = \mathcal{O}_i^{\text{bs}} \cup \{j : y_{ij} = 1, j \in \mathcal{C}_i^{\text{bs}}\}.$$

Then, (1) can be rewritten as:

$$\beta_i + \sum_{j \in \mathcal{O}_i^{\text{bs}}} \alpha_{ij} + \sum_{j \in \mathcal{C}_i^{\text{bs}}} \alpha_{ij} \cdot y_{ij} = \sigma_i, \quad i \in \mathcal{N}. \tag{11}$$

Likewise, for (2), we need to first clarify $\mathcal{I}_j^{\text{usr}}$, i.e., the set of users that are interfering with BS j. Based on the definitions of $\mathcal{O}_j^{\text{usr}}$, $\mathcal{C}_j^{\text{usr}}$, and y_{ij}, we have

$$\mathcal{I}_j^{\text{usr}} = \mathcal{O}_j^{\text{usr}} \cup \{i : y_{ij} = 1, i \in \mathcal{C}_j^{\text{usr}}\}. \tag{12}$$

Depending on whether user i in $\mathcal{O}_j^{\text{usr}}$ or $\mathcal{C}_j^{\text{usr}}$, (2) can be rewritten as:

$$\alpha_{ij} \leq \sum_{k \in \mathcal{O}_j^{\text{usr}}}^{k \neq i} \beta_k + \sum_{k \in \mathcal{C}_j^{\text{usr}}} \beta_k \cdot y_{kj}, \quad i \in \mathcal{O}_j^{\text{usr}}, j \in \mathcal{M}, \tag{13}$$

$$\alpha_{ij} \cdot y_{ij} \leq \sum_{k \in \mathcal{O}_j^{\text{usr}}} \beta_k + \sum_{k \in \mathcal{C}_j^{\text{usr}}}^{k \neq i} \beta_k \cdot y_{kj}, \quad i \in \mathcal{C}_j^{\text{usr}}, j \in \mathcal{M}, \tag{14}$$

Finally, for (3), we need to first clarify $\mathcal{T}_j^{\text{usr}}$, i.e., the set of users that choose BS j as their service provider. Based on the definitions of $\mathcal{C}_j^{\text{usr}}$ and x_{ij}, we have

$$\mathcal{T}_j^{\text{usr}} = \{i : x_{ij} = 1, i \in \mathcal{C}_j^{\text{usr}}\}.$$

Then, (3) can be rewritten as:

$$\sum_{i \in \mathcal{C}_j^{\text{usr}}} \sigma_i \cdot x_{ij} + \sum_{i \in \mathcal{I}_j^{\text{usr}}} (\sigma_i - \alpha_{ij}) \leq K \qquad j \in \mathcal{M} .$$

which is equivalent to

$$\sum_{i \in \mathcal{C}_j^{\text{usr}}} \sigma_i \cdot x_{ij} + \sum_{i \in \mathcal{C}_j^{\text{usr}}} (\sigma_i - \alpha_{ij}) \cdot y_{ij} + \sum_{i \in \mathcal{O}_j^{\text{usr}}} (\sigma_i - \alpha_{ij}) \leq K, \; j \in \mathcal{M}, \tag{15}$$

based on $\mathcal{I}_j^{\text{usr}}$ in (12).

Constraints (9)–(15) define a feasible IA design space when user scheduling is jointly considered. In the sequel, we employ this IA space to study an uplink user throughput maximization problem in a cellular network.

B. *User Throughput Optimization Framework*

For simplicity, we assume that fixed modulation and coding scheme (MCS) is used for each data stream and that each data stream corresponds to one unit data rate. The goal is to maximize the minimum rate among all the users. Denote $r_{\min}$ as the minimum rate among all users. Then we have

$$\sigma_i \geq r_{\min} , \quad i \in \mathcal{N}. \tag{16}$$

Based on the constraints in Section V-A, the user throughput maximization problem can be formulated as follows:

OPT-IA$^{\text{raw}}$:	Max	$r_{\min}$
	S.t.	User scheduling: (9), (10);
		IA constraints: (11), (13), (14), (15);
		Minimum rate constraints: (16).

OPT-IA$^{\text{raw}}$ is a mixed integer nonlinear programming (MINLP). To eliminate the nonlinear terms in the constraints, we employ the *Reformulation-Linearization Technique (RLT)* in [28]. Specifically, to eliminate the nonlinear term $\alpha_{ij} \cdot y_{ij}$ in the constraints, we define $\lambda_{ij} = \alpha_{ij} \cdot y_{ij}$. This replacement requires to add the following two constraints:

$$0 \leq \lambda_{ij} \leq \alpha_{ij}, \quad j \in \mathcal{C}_i^{\text{bs}}, i \in \mathcal{N}, \tag{17}$$

$$\alpha_{ij} - (1 - y_{ij}) \cdot K \leq \lambda_{ij} \leq y_{ij} \cdot K, \quad j \in \mathcal{C}_i^{\text{bs}}, i \in \mathcal{N}. \tag{18}$$

Similarly, to eliminate the nonlinear term $\beta_i \cdot y_{ij}$ in the constraints, we define $\mu_{ij} = \beta_i \cdot y_{ij}$. This replacement requires to add the following two constraints:

$$0 \leq \mu_{ij} \leq \beta_i, \quad j \in \mathcal{C}_i^{\text{bs}}, i \in \mathcal{N}, \tag{19}$$

$$\beta_i - (1 - y_{ij}) \cdot K \leq \mu_{ij} \leq y_{ij} \cdot K, \quad j \in \mathcal{C}_i^{\text{bs}}, i \in \mathcal{N}. \tag{20}$$

By replacing $\lambda_{ij} = \alpha_{ij} \cdot y_{ij}$ and $\mu_{ij} = \beta_i \cdot y_{ij}$ in the IA constraints (11), (13), (14), (15), we have the following linear IA constraints:

$$\beta_i + \sum_{j \in \mathcal{O}_i^{\text{bs}}} \alpha_{ij} + \sum_{j \in \mathcal{C}_i^{\text{bs}}} \lambda_{ij} = \sigma_i, \quad i \in \mathcal{N}, \tag{21}$$

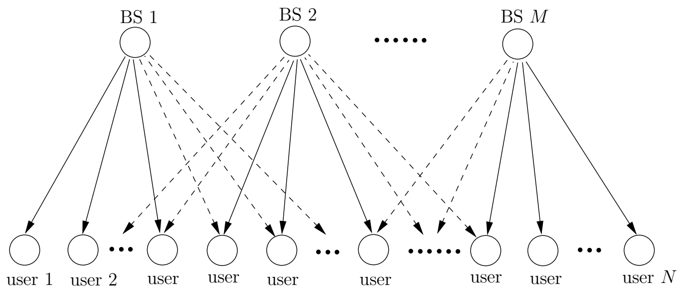


Fig. 6: Downlink communication in a cellular network.

$$\alpha_{ij} \leq \sum_{k\in\mathcal{O}_j^{\text{usr}}}^{k\neq i} \beta_k + \sum_{k\in\mathcal{C}_j^{\text{usr}}} \mu_{kj}, \quad i \in \mathcal{O}_j^{\text{usr}}, j \in \mathcal{M}, \tag{22}$$

$$\lambda_{ij} \leq \sum_{k\in\mathcal{O}_j^{\text{usr}}} \beta_k + \sum_{k\in\mathcal{C}_j^{\text{usr}}}^{k\neq i} \mu_{kj}, \quad i \in \mathcal{C}_j^{\text{usr}}, j \in \mathcal{M}, \tag{23}$$

$$\sum_{i\in\mathcal{C}_j^{\text{usr}}} (\sigma_i - \lambda_{ij}) + \sum_{i\in\mathcal{O}_j^{\text{usr}}} (\sigma_i - \alpha_{ij}) \leq K, \quad j \in \mathcal{M}. \tag{24}$$

Then, OPT-IA$^{\text{raw}}$ is reformulated as follows:

OPT-IA:	Max	$r_{\min}$
	S.t.	User scheduling: (9), (10);
		IA constraints: (17), (18), (19), (20), (21), (22), (23), (24);
		Minimum rate constraints: (16);

where $\mathcal{N}$, $\mathcal{M}$, $\mathcal{C}_i^{\text{bs}}$, $\mathcal{O}_i^{\text{bs}}$, $\mathcal{C}_j^{\text{usr}}$, $\mathcal{O}_j^{\text{usr}}$, and K are known; x_{ij} and y_{ij} are binary variables; $r_{\min}$, σ_i, α_{ij}, β_i, λ_{ij}, and μ_{ij} are non-negative integer variables.

OPT-IA is a mixed integer linear programming (MILP). Although the theoretical worst-case complexity to a general MILP problem is exponential [29], [30], there exist highly efficient optimality/approximation algorithms (e.g., branch-and-bound with cutting planes [31]) and heuristics (e.g., sequential fixing algorithm [32]). Another approach is to employ an off-the-shelf solver such as IBM CPLEX optimization solver [33], which can successfully handle a moderate-sized network. As the main goal of this paper is to study IA from a networking perspective rather than developing a specific solution to an optimization problem, we will employ the IBM CPLEX optimization solver to obtain numerical results in the next section.

VI. Duality Between Uplink and Downlink

In the previous sections, we studied an IA scheme for the uplink of a cellular network. We now consider the downlink case. We will show that the IA scheme developed for uplink can also be applied to downlink and therefore the downlink user throughput maximization problem can be solved in the same way as the uplink problem.

Consider the downlink communication as shown in Fig. 6, which has the same setting as the uplink in Fig. 3. For the downlink, denote $\hat{\pi}$ as its IA scheme with DoF vector $(\hat{\sigma}_1, \hat{\sigma}_2, \cdots, \hat{\sigma}_N)$, where $\hat{\sigma}_i$ is the number of desired data streams at user i. At user i, denote $\hat{\mathcal{S}}_i = \{\hat{s}_i^k : 1 \leq k \leq \hat{\sigma}_i\}$ as the set of its desired data streams and $\hat{\mathbf{v}}_i^k$ as the decoding vector of its stream $\hat{s}_i^k$. At user i's intended BS j, denote $\hat{\mathbf{u}}_j^l$ as the precoding vector of user i's stream $\hat{s}_i^k$. Then we have the following theorem:

Theorem 2: For downlink IA scheme $\hat{\pi}$, if its DoF vector $(\hat{\sigma}_1, \hat{\sigma}_2, \cdots, \hat{\sigma}_N)$ satisfies (1), (2), and (3), then it is feasible at the physical layer.

A proof of Theorem 2 is given in Appendix D. Based on Theorem 2, we have the following observations on π and $\hat{\pi}$:

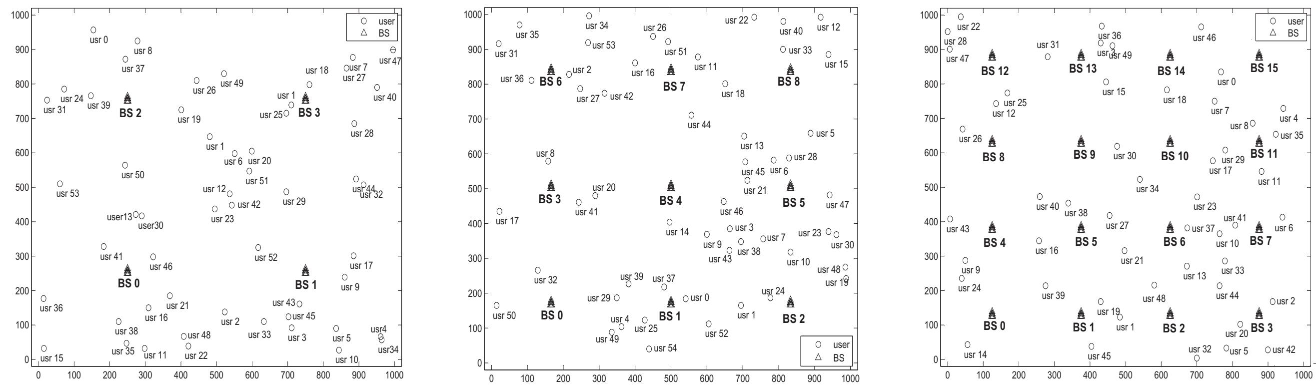
- For user $i \in \mathcal{N}$, if it can send σ_i data streams to its BS j in the uplink transmission, then it can receive σ_i data streams from BS j in the downlink transmission, and vice versa.
- For the downlink problem, the precoding and decoding vectors in $\hat{\pi}$ are the same as the corresponding decoding and precoding vectors in π, respectively. That is, stream $\hat{s}_i^k$'s precoding vector is stream s_i^k's decoding vector and stream $\hat{s}_i^k$'s decoding vector is stream s_i^k's precoding vector.
- Since the uplink and downlink have the same IA design space, the downlink user throughput maximization problem has the same formulation as the uplink problem. Therefore, the downlink has the same optimal user throughput as the uplink.

VII. Performance Evaluation

In this section, we first use a case study to illustrate how IA scheme works in a cellular network. Then, we compare the user throughput performance of our IA scheme against two other schemes: "no-IA" scheme and "crude-IA" scheme. In no-IA scheme, a subset of subcarriers is allocated to each user for its data transmission, with each data or interfering stream occupying a unique subcarrier at each BS. That is, there is a complete absence of overlapping of interfering streams on any subcarrier. We denote the user throughput maximization problem under no-IA scheme as OPT-noIA and its formulation is given in Appendix E. In crude-IA scheme, a subset of subcarriers is allocated to each user for its data transmission so that at a BS, each of its desired data streams is on a unique subcarrier while the interfering streams are allowed to overlap. This problem is similar to ours except that each data stream in our IA scheme is projected onto all subcarriers and there is an optimization on the design of directions for intended data streams and interfering data streams. In light of this key difference, we denote the user throughput maximization problem under crude-IA scheme as OPT-crudeIA and its formulation is given in Appendix F.

A. Simulation Setting

Without loss of generality, we normalize all units for distance, time, bandwidth, and data rate with appropriate dimensions. We consider cellular networks within a 1000×1000 area for three cases: (i) 4 BSs with 100 users; (ii) 9 BSs with 100 users; and (iii) 16 BSs with 100 users. Fig. 7 illustrates our BS deployment for the three cases. In each case, to mimic



(a) A network instance with 4 BSs (b) A network instance with 9 BSs (c) A network instance with 16 BSs

Fig. 7: Cellular network instances with 4, 9, and 16 BSs.

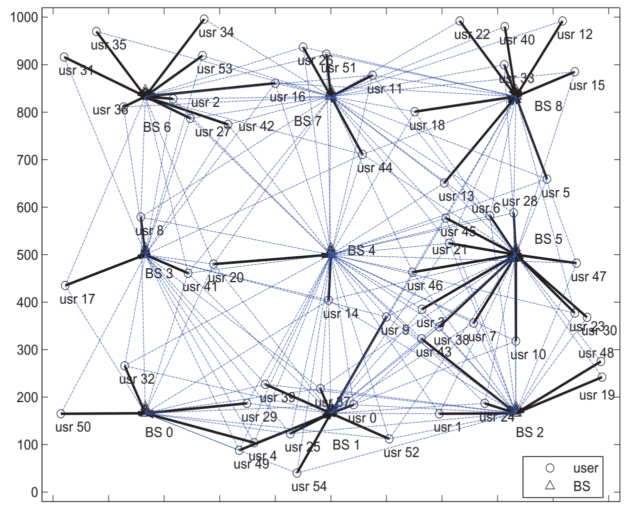


Fig. 8: User scheduling at each BS and interference pattern.

the BS deployment in real-world cellular networks, we place the BSs in grid. The 100 users are randomly distributed in the area with a uniform probability. A user can be in "active" or "inactive" state, with equal probability. When active, a user has a persistent traffic for transmission; when inactive, a user is not served by any BS. For any comparison study, the state of a user is the same under all three schemes. To assure that each user is within the service area of at least one BS, we set the transmission range to 360 for the 4-BS case, 240 for the 9-BS case, and 180 for the 16-BS case. The interference range is twice of the transmission range and the number of subcarriers available for data transmission is 256, unless otherwise specified.

B. A Case Study

We use the network instance in Fig. 7(b) to illustrate how IA works to improve throughput. Among the 100 users, 55 of them are active and 45 of them are inactive (inactive users are not shown in the figure). The number of subcarriers available for IA is 256. By solving the OPT-IA problem for this network instance, we obtain the optimal objective value of 13. We then solve the OPT-noIA problem for this network instance, we obtain the optimal objective value of 6. This indicates that our IA scheme can increase the user throughput by 117% as compared to the no-IA scheme. We also solve the OPT-crudeIA problem for this network instance, we obtain the optimal objective value of 9. This indicates that our IA scheme can increase the user throughput by 44% as compared to the crude-IA scheme.

We now show how our IA scheme works in this network instance. Fig. 8 shows the user scheduling result and interference pattern in the IA solution, where a solid arrow line represents an established link from a user to a BS and a dashed line represents an interference. Table II summarizes the IA behavior at each BS. In this table, the first column lists the BSs in the network; the second column lists the number of users that choose this BS as their service provider; the third column lists the number of desired data streams at this BS, where each user has 13 data streams to its BS; the fourth column lists the dimension of the subspace for the interfering streams at this BS, which is 256 minus the number in the third column; the fifth column lists the number of undesired interfering streams (from neighboring interfering users) at this BS; the sixth column lists the interference overlapping ratio, which is the ratio of the fifth column to the fourth column. In the sixth column, a value greater than 1 indicates the existence of interference overlapping. The large the ratio is, the more IA has been achieved at the corresponding BS.

Now let's take a look at the row for BS 5 in Table II as an example. As shown in Fig. 8, BS 5 is used as service provider by 12 users. Since each user has 13 outgoing data streams, the number of desired data streams at BS 5 is 156. Thus, the dimension of the subspace for the interfering streams is upper bounded by 100 (i.e., $256 - 156$). As shown in Fig. 8, BS 5 is being interfered by 17 users and thus has 221 (i.e., 17×13) interfering streams. Therefore, the interference overlapping ratio at BS 5 is $221/100 = 2.21$ (as shown in the table). As indicated in Table II, interfering streams are squeezed in a reduced-dimensional subspace at most BSs in this network instance, making it possible to transport more data streams than the no-IA and crude-IA schemes.

TABLE II: IA behavior at each BS in the case study.

BS j	# of users	# of data streams	dimension of interfering streams	# of interfering streams	Interference overlapping ratio
BS 0	4	52	204	156	0.76
BS 1	8	104	152	208	1.37
BS 2	5	65	191	260	1.36
BS 3	3	39	217	247	1.14
BS 4	2	26	230	494	2.15
BS 5	12	156	100	221	2.21
BS 6	9	117	139	104	0.75
BS 7	4	52	204	325	1.59
BS 8	8	104	152	156	1.03

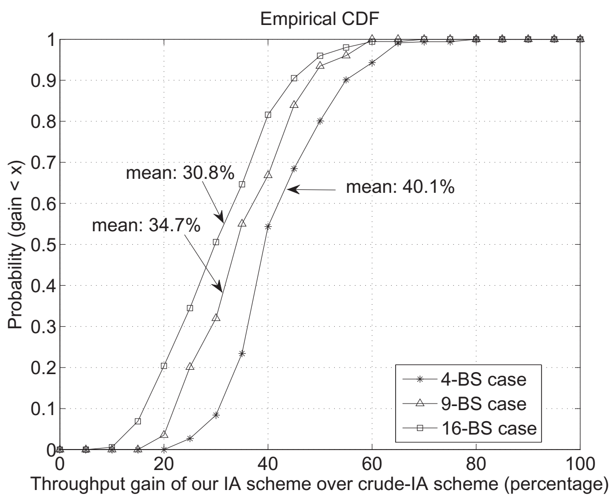


Fig. 9: Throughput gain of our IA scheme over crude-IA scheme.

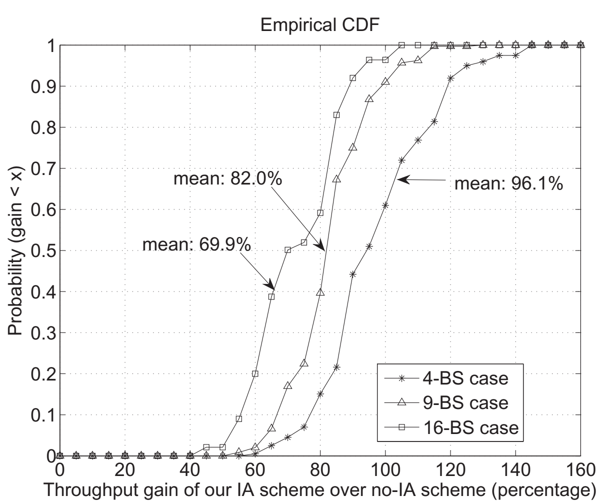


Fig. 10: Throughput gain of our IA scheme over no-IA scheme.

C. Throughput Gain of Our IA Scheme

To study throughput gain of our IA scheme over crude-IA and no-IA schemes, we generate 200 randomly network instances with 256 subcarriers. For each network instance, we solve its OPT-IA, OPT-crudeIA, and OPT-noIA formulations

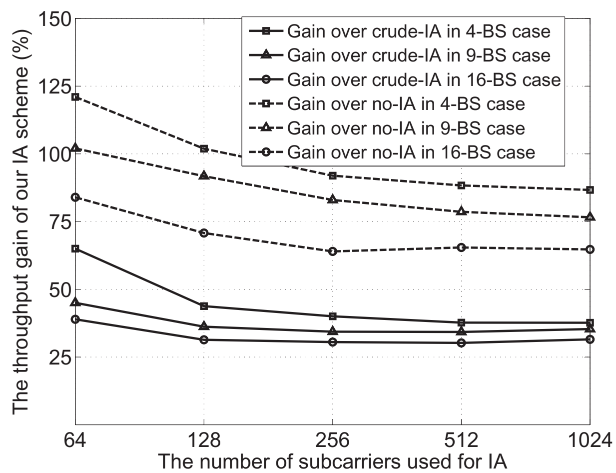


Fig. 11: Impact of subcarrier number on the gain of our IA scheme.

using CPLEX optimization solver and obtain their optimal objective values. Fig. 9 presents the throughput gain of our IA scheme over crude-IA scheme in three cases (4-BS, 9-BS, 16-BS), where x-axis is the throughput gain in percentage (i.e. the ratio of the optimal objective value from OPT-IA to that from OPT-crudeIA minus one and times 100) and the y-axis is the cumulative probability. From the figure we can see that the throughput gain ranges from 10% to 70% in the three cases, indicating that our IA scheme always outperforms crude-IA scheme. On average, the throughput gain of our IA scheme over crude-IA scheme is 40.1% in the 4-BS case, 34.7% in the 9-BS case, and 30.8% in the 16-BS case. In the same format, Fig. 10 presents the throughput gain of our IA scheme over no-IA scheme. On average, the throughput gain of our IA scheme over no-IA scheme is 96.1% in the 4-BS case, 82.0% in the 9-BS case, and 69.9% in the 16-BS case. From Fig. 9 and Fig. 10, we can see that our IA scheme has higher throughput gain over no-IA scheme than over crude-IA scheme. This is not surprising, as the no-IA scheme does not allow any alignment whereas the crude-IA scheme allows alignment of interference on each individual subcarrier.

Impact from The Number of Subcarriers. We now study the impact of subcarrier number on the throughput gain of our IA scheme. We generate 200 randomly network instances with the number of subcarriers varying from 32, 64, 128, 256, 512, to 1024. Fig. 11 presents the throughput gain of our IA

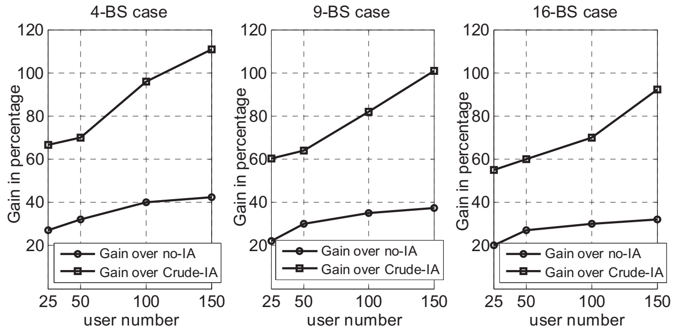


Fig. 12: Impact of user density on the gain of our IA scheme.

scheme (averaged over the 200 network instances) versus the number of subcarriers in 4-BS, 9-BS, and 16-BS cases. For example, when the network has 4 BSs and 64 subcarriers, the average throughput gain of our IA scheme is 65% over crude-IA scheme and 121% over no-IA scheme, as shown in Fig. 11. From the figure we can see that, as the number of subcarriers increases, the throughput gains of our IA scheme over crude-IA scheme converge to 37.6%, 35.3%, 31.5% in 4-BS, 9-BS, and 16-BS cases; and the throughput gains of IA scheme over no-IA scheme converge to 86.7%, 76.6%, 64.7% in the three cases. It should be noted that when the number of subcarriers is less than 64, crude-IA and no-IA schemes yield zero throughput for most network instances.

Impact of User Density. Finally, we study the impact of user density on the throughput gain of our IA scheme over no-IA and crude-IA schemes. For each network instance (see, e.g., Fig. 7), instead of fixing user number to 100, we consider different user densities: 25 users, 50 users, 100 users, and 150 users (within the 1000×1000 square area). For each user density, we generate 200 network instances and compute their averaged gain of our IA scheme. Fig. 12 presents the throughput gain of our IA scheme in 4-BS, 9-BS, and 16-BS cases, where x-axis is the number of users in the 1000×1000 square area and y-axis is the averaged gain of our IA scheme over the 200 network instances. From the figure we can see that the gain of our IA scheme over both no-IA and crude-IA schemes becomes more significant as the number of users increases. As we explained in Section III, this is because more users can achieve more alignment at each BS, thereby leaving larger subspace available for desired data stream reception at each BS. Therefore, our IA scheme is more suitable for a dense network.

VIII. Conclusions and Future Work

This paper studied IA in cellular networks from a networking perspective. We developed an OFDM-based IA scheme for cellular networks and proved its feasibility at the physical layer. Specifically, we showed that as long as the corresponding IA constraints are satisfied, there always exist precoding and decoding vectors so that each data stream can be transported free of interference. Such an IA scheme allows us to study network-level throughput problems without getting involved into the onerous design of precoding and decoding vectors. By incorporating user scheduling into our IA scheme, we developed an uplink user throughput optimization framework and demonstrated the throughput gain of our IA scheme at network level. For the downlink problem, we showed that the IA scheme developed for the uplink can also be applied to the downlink. Furthermore, the downlink user throughput maximization problem has the same formulation as the uplink problem and therefore can be solved in the same way. Although the IA scheme was designed in the frequency domain, it is a general framework which can also be used in the temporal and spatial domains.

While the benefit of IA has been recognized in theory, there are a number of issues needed to be addressed in order to use it in practical cellular networks, including CSI acquisition on the transmitter side, transmission/reception coordination among the nodes in the network, and timing and frequency synchronization among the transmitters. Obviously, these issues will compromise the throughput gain of the proposed IA scheme. In our future work, we will develop practical frequency-domain IA solutions that can address those issues while maximally preserving the throughput gain of IA in real-world cellular networks.

Appendix A
Proof of Lemma 1

Based on the definitions of $\mathcal{E}^{S_i}$, $\mathcal{E}^{B_i}$, and $\mathcal{E}^{A_{ij}}$, we have $\mathcal{E}^{S_i} = \mathcal{E}^{B_i} \cup (\cup_{j \in \mathcal{I}_i^{\text{bs}}} \mathcal{E}^{A_{ij}})$. According to the precoding vector construction procedure, we know that the constructed precoding vectors in $\mathcal{E}^{B_i}$ are independent of any channel matrices (see (6)), whereas the constructed precoding vectors in $\mathcal{E}^{A_{ij}}$ are determined by the channel matrices (see (7)). Given that the diagonal entries in the channel matrices are drawn from complex Gaussian distribution, we have

$$\begin{aligned}\dim(\mathcal{E}^{S_i}) &= \dim(\mathcal{E}^{B_i} \cup (\cup_{j \in \mathcal{I}_i^{\text{bs}}} \mathcal{E}^{A_{ij}})) \\ &= \dim(\mathcal{E}^{B_i}) + \dim(\cup_{j \in \mathcal{I}_i^{\text{bs}}} \mathcal{E}^{A_{ij}}).\end{aligned} \tag{25}$$

Based on (7), we know that the precoding vectors in $\mathcal{E}^{A_{ij}}$ is determined by the channel matrix $\mathbf{H}_{ji}$. Since the channel matrices in $\{\mathbf{H}_{ji} : j \in \mathcal{I}_i^{\text{bs}}\}$ are randomly independent of each other, we have

$$\dim(\cup_{j \in \mathcal{I}_i^{\text{bs}}} \mathcal{E}^{A_{ij}}) = \sum_{j \in \mathcal{I}_i^{\text{bs}}} \dim(\mathcal{E}^{A_{ij}}). \tag{26}$$

To analyze $\dim(\mathcal{E}^{A_{ij}})$, we divide the precoding vectors in $\mathcal{E}^{A_{ij}}$ into different groups based on the their corresponding value of i' in (7): $\{\mathcal{E}^{A_{ij}^{i'}} : i' \in \mathcal{I}_j^{\text{usr}}, i' \neq i\}$. Thus we have $\mathcal{E}^{A_{ij}} = \cup_{i' \in \mathcal{I}_j^{\text{usr}}}^{i' \neq i} \mathcal{E}^{A_{ij}^{i'}}$, where $\mathcal{E}^{A_{ij}^{i'}} := \mathbf{H}_{ji}^{-1} \mathbf{H}_{ji'} \mathcal{E}^{\tilde{B}_{i'}}$ with $\mathcal{E}^{\tilde{B}_{i'}} \subseteq \mathcal{E}^{B_{i'}}$. Based on the precoding vector construction procedure, we have

$$\dim(\mathcal{E}^{A_{ij}^{i'}}) \overset{(a)}{=} \dim(\mathcal{E}^{B_{i'}}) \overset{(b)}{=} |\mathcal{E}^{B_{i'}}| \overset{(c)}{=} |\mathcal{E}^{A_{ij}^{i'}}|, \tag{27}$$

where (a) follows from our mild assumption that channel matrix has full rank [34, Ch. 1]; (b) follows from the fact that the precoding vectors in $\mathcal{E}^{B_{i'}}$ are constructed by (6); (c) follows from the fact that in our IA scheme, each interfering streams in $\mathcal{A}_{ij}$ is aligned to a unique interfering stream in $\mathcal{B}_{i'}$ with $i' \neq i$.

Based on the definitions and (27), we have

$$\dim(\mathcal{E}^{A_{ij}}) = \dim(\cup_{i' \in \mathcal{I}_j^{\text{usr}}}^{i' \neq i} \mathcal{E}^{A_{ij}^{i'}}) \overset{(a)}{=} \sum_{i' \in \mathcal{I}_j^{\text{usr}}}^{i' \neq i} \dim(\mathcal{E}^{A_{ij}^{i'}}) = \sum_{i' \in \mathcal{I}_j^{\text{usr}}}^{i' \neq i} |\mathcal{E}^{A_{ij}^{i'}}| = |\mathcal{E}^{A_{ij}}|, \quad (28)$$

where (a) follows from the fact that the channel matrices $\{\mathbf{H}_{ji'} : i' \in \mathcal{I}_j^{\text{usr}}, i' \neq i\}$ are drawn complex Gaussian distribution and independent of each other.

Based on (25), (26), and (28), we conclude

$$\begin{aligned}\dim(\mathcal{E}^{S_i}) &= \dim(\mathcal{E}^{B_i}) + \dim(\cup_{j \in \mathcal{I}_i^{\text{bs}}} \mathcal{E}^{A_{ij}}),\\ &= |\mathcal{E}^{B_i}| + \sum_{j \in \mathcal{I}_i^{\text{bs}}} \dim(\mathcal{E}^{A_{ij}})\\ &= |\mathcal{E}^{B_i}| + \sum_{j \in \mathcal{I}_i^{\text{bs}}} |\mathcal{E}^{A_{ij}}|\\ &= |\mathcal{E}^{S_i}|.\end{aligned}$$

This complete the proof.

Appendix B
Proof of Lemma 2

Consider a BS $j \in \mathcal{M}$ as shown in Fig. 4(b). Denote $\mathcal{Q}_j^{\text{I,Ali}}$ as the set of aligned interfering stream directions. Denote $\mathcal{Q}_j^{\text{I,Def}}$ as the set of predefined interfering stream directions. Denote $\mathcal{Q}_j^{\text{I,Eff}}$ as the set of "effective" interfering stream directions. Then we have

$$\begin{aligned}\mathcal{Q}_j^{\text{I,Ali}} &= \cup_{i \in \mathcal{I}_j^{\text{usr}}} \{\mathbf{H}_{ji}\mathbf{u}_i^k : \mathbf{u}_i^k \in \mathcal{E}^{A_{ij}}\},\\ \mathcal{Q}_j^{\text{I,Def}} &= \cup_{i \in \mathcal{I}_j^{\text{usr}}} \{\mathbf{H}_{ji}\mathbf{u}_i^k : \mathbf{u}_i^k \in \mathcal{E}^{B_i}\},\\ \mathcal{Q}_j^{\text{I,Eff}} &= \cup_{i \in \mathcal{I}_j^{\text{usr}}} \{\mathbf{H}_{ji}\mathbf{u}_i^k : \mathbf{u}_i^k \in \mathcal{E}^{S_i} \backslash \mathcal{E}^{A_{ij}}\}.\end{aligned}$$

Since $\mathcal{E}^{B_i} \subseteq \mathcal{E}^{S_i} \backslash \mathcal{E}^{A_{ij}}$, we have $\mathcal{Q}_j^{\text{I,Def}} \subseteq \mathcal{Q}_j^{\text{I,Eff}}$. Based on the precoding vector construction procedure, we know that for each $\mathbf{H}_{ji}\mathbf{u}_i^k \in \mathcal{Q}_j^{\text{I,Ali}}$, there exists a $\mathbf{H}_{ji'}\mathbf{u}_{i'}^{k'} \in \mathcal{Q}_j^{\text{I,Eff}}$ such that $\mathbf{H}_{ji}\mathbf{u}_i^k := \mathbf{H}_{ji'}\mathbf{u}_{i'}^{k'}$. Consequentially, we have $\text{span}(\mathcal{Q}_j^{\text{I,Ali}}) \subseteq \text{span}(\mathcal{Q}_j^{\text{I,Def}})$. Thus we have

$$\text{span}(\mathcal{Q}_j^{\text{I}}) = \text{span}(\mathcal{Q}_j^{\text{I,Eff}} \cup \mathcal{Q}_j^{\text{I,Ali}}) = \text{span}(\mathcal{Q}_j^{\text{I,Eff}}). \quad (29)$$

We now argue that the signal subspace $\mathcal{Q}_j^{\text{T}}$ is linearly independent of the "effective" interference subspace $\mathcal{Q}_j^{\text{I,Eff}}$ at BS j. This is true for the following two reasons. First, based on the given constraint (3), we have $|\mathcal{Q}_j^{\text{T}} \cup \mathcal{Q}_j^{\text{I,Eff}}| = \sum_{i \in \mathcal{T}_j^{\text{usr}}} \sigma_i + \sum_{i \in \mathcal{I}_j^{\text{usr}}} (\sigma_i - \alpha_{ij}) \leq K$. Thus, the number of directions in $\mathcal{Q}_j^{\text{T}} \cup \mathcal{Q}_j^{\text{I,Eff}}$ is bounded by the total available dimension (i.e., the number of subcarriers K). Second, the channel matrices $\{\mathbf{H}_{ji} : i \in \mathcal{T}_j^{\text{usr}} \cup \mathcal{I}_j^{\text{usr}}\}$ are frequency-selective and are randomly independent of each other. These properties of the channel matrices are attributive to the network environment. For these two reasons, we have

$$\dim(\mathcal{Q}_j^{\text{T}} \cup \mathcal{Q}_j^{\text{I,Eff}}) = \dim(\mathcal{Q}_j^{\text{T}}) + \dim(\mathcal{Q}_j^{\text{I,Eff}}). \quad (30)$$

To characterize the dimension of the signal (desired data stream) subspace at BS j, we have

$$\begin{aligned}\dim(\mathcal{Q}_j^{\text{T}}) &= \dim(\cup_{i \in \mathcal{T}_j^{\text{usr}}} \{\mathbf{H}_{ji}\mathbf{u}_i^k : \mathbf{u}_i^k \in \mathcal{E}^{S_i}\})\\ &\overset{(a)}{=} \sum_{i \in \mathcal{T}_j^{\text{usr}}} \dim(\{\mathbf{H}_{ji}\mathbf{u}_i^k : \mathbf{u}_i^k \in \mathcal{E}^{S_i}\})\\ &\overset{(b)}{=} \sum_{i \in \mathcal{T}_j^{\text{usr}}} \dim(\mathcal{E}^{S_i})\\ &\overset{(c)}{=} \sum_{i \in \mathcal{T}_j^{\text{usr}}} \sigma_i, \end{aligned} \quad (31)$$

where (a) holds due to the random independence of the channel matrices $\{\mathbf{H}_{ji} : i \in \mathcal{T}_j^{\text{usr}}\}$ and $|\mathcal{Q}_j^{\text{T}}| \leq K$; (b) follows from our mild assumption that $\mathbf{H}_{ji}$ has full rank; (c) follows from Lemma 1 and $|\mathcal{E}^{S_i}| = \sigma_i$.

Based on (29) and (31), we have

$$\begin{aligned}\dim(\mathcal{Q}_j^{\text{T}} \cup \mathcal{Q}_j^{\text{I}}) &\overset{(a)}{=} \dim(\mathcal{Q}_j^{\text{T}} \cup \mathcal{Q}_j^{\text{I,Eff}})\\ &\overset{(b)}{=} \dim(\mathcal{Q}_j^{\text{T}}) + \dim(\mathcal{Q}_j^{\text{I,Eff}})\\ &\overset{(c)}{=} \sum_{i \in \mathcal{T}_j^{\text{usr}}} \sigma_i + \dim(\mathcal{Q}_j^{\text{I}}), \end{aligned} \quad (32)$$

where (a) and (c) hold due to (29); (b) holds due to (31).

Combining (32) and Lemma 1, we conclude that Theorem 1 holds. This completes the proof.

Appendix C
Proof of Proposition 1

We show that if the precoding vectors satisfy constraint (8), then there exist a set of decoding vectors that satisfy (4) and (5) in Definition 1. Specifically, we argue that if constraint (8) is satisfied, then the following linear system is consistent (i.e., the system has at least one feasible solution):

$$\begin{aligned}&(\mathbf{v}_j^l)^T \mathbf{H}_{ji}\mathbf{u}_i^k = 1,\\ &(\mathbf{v}_j^l)^T \mathbf{H}_{ji'}\mathbf{u}_{i'}^{k'} = 0, \ i' \in \mathcal{T}_j^{\text{usr}} \cup \mathcal{I}_j^{\text{usr}}, 1 \leq k' \leq \sigma_{i'}, (i', k') \neq (i, k)\end{aligned}$$

where $\mathbf{v}_j^l$ is variable vector while $\mathbf{H}$'s and $\mathbf{u}$'s are known.

Based on the definition of $\mathcal{Q}_j^{\text{T}}$ and $\mathcal{Q}_j^{\text{I}}$, we know

$$\mathcal{Q}_j^{\text{T}} \cup \mathcal{Q}_j^{\text{I}} = \{\mathbf{H}_{ji'}\mathbf{u}_{i'}^{k'} : i' \in \mathcal{T}_j^{\text{usr}} \cup \mathcal{I}_j^{\text{usr}}, 1 \leq k' \leq \sigma_{i'}\}.$$

It is easy to see that $\mathcal{Q}_j^{\text{T}} \cup \mathcal{Q}_j^{\text{I}}$ is the set of coefficient-vectors of this linear system. Moreover, this system has K free variables and at most K linearly independent equations. If we can show that vector $\mathbf{H}_{ji}\mathbf{u}_i^k$ is not a linear combination of other vectors in $\mathcal{Q}_j^{\text{T}} \cup \mathcal{Q}_j^{\text{I}}$, then this system is consistent. Next, we argue this point by contradiction.

Suppose that $\mathbf{H}_{ji}\mathbf{u}_i^k$ is a linear combination of other vectors in $\mathcal{Q}_j^{\text{T}} \cup \mathcal{Q}_j^{\text{I}}$. Given $\mathbf{H}_{ji}\mathbf{u}_i^k \in \mathcal{Q}_j^{\text{T}}$, we have

$$\dim(\mathcal{Q}_j^{\text{T}} \cup \mathcal{Q}_j^{\text{I}}) < |\mathcal{Q}_j^{\text{T}}| + \dim(\mathcal{Q}_j^{\text{I}}) = \sum_{i \in \mathcal{I}_j^{\text{usr}}} \sigma_i + \dim(\mathcal{Q}_j^{\text{I}}).$$

This contradicts the given condition in (8). Therefore, we conclude that the linear system is consistent. This completes the proof.

APPENDIX D
PROOF OF THEOREM 2

We prove it by construction. Consider an uplink IA scheme π with its DoF vector $(\sigma_1, \sigma_2, \cdots, \sigma_N) = (\hat{\sigma}_1, \hat{\sigma}_2, \cdots, \hat{\sigma}_N)$. Since $(\hat{\sigma}_1, \hat{\sigma}_2, \cdots, \hat{\sigma}_N)$ satisfies (1), (2), and (3), based on Theorem 1, we know that the uplink IA scheme π is feasible. Further, for each stream s_i^k in uplink IA scheme π, there exist a precoding vector $\mathbf{u}_i^k$ and a decoding vector $\mathbf{v}_j^l$ that satisfy (4) and (5).

To show that downlink IA scheme $\hat{\pi}$ is feasible, we construct each stream $\hat{s}_i^k$'s precoding and decoding vectors as follows: $\hat{\mathbf{u}}_j^l = \mathbf{v}_j^l$ and $\hat{\mathbf{v}}_i^k = \mathbf{u}_i^k$, where $\mathbf{v}_j^l$ and $\mathbf{u}_i^k$ are decoding and precoding vectors in the uplink IA scheme π and can been constructed in (6) and (7). Now we argue that by using these precoding and decoding vectors, each steam $\hat{s}_i^k$ in downlink IA scheme $\hat{\pi}$ can be transported free of interference.

We first check the transfer function of stream $\hat{s}_i^k$ as follows:

$$(\hat{\mathbf{v}}_i^k)^T \mathbf{H}_{ji} \hat{\mathbf{u}}_j^l \overset{(a)}{=} (\mathbf{u}_i^k)^T \mathbf{H}_{ji} \mathbf{v}_j^l \overset{(b)}{=} [(\mathbf{v}_j^l)^T \mathbf{H}_{ji} \mathbf{u}_i^k]^T \overset{(c)}{=} 1, \quad (33)$$

where (a) follows from $\hat{\mathbf{u}}_j^l = \mathbf{v}_j^l$ and $\hat{\mathbf{v}}_i^k = \mathbf{u}_i^k$; (b) follows from the fact that $\mathbf{H}_{ji}$ is a diagonal matrix, i.e., $(\mathbf{H}_{ji})^T = \mathbf{H}_{ji}$; and (c) follows from (4).

We then check whether the interference can be completely canceled. For stream $\hat{s}_i^k$, it suffers from interference from the streams that correspond to precoding vectors $\mathbf{v}_{j'}^{l'}$ with $j' \in \mathcal{I}_i^{\text{bs}} \cup \{j\}$, $1 \le l' \le \sum_{i' \in \mathcal{T}_{j'}} \sigma_{i'}$, and $(j', l') \neq (j, l)$. Based on (5), we have

$$(\hat{\mathbf{v}}_i^k)^T \mathbf{H}_{ji} \hat{\mathbf{u}}_{j'}^{l'} = (\mathbf{u}_i^k)^T \mathbf{H}_{ji} \mathbf{v}_{j'}^{l'} = [(\mathbf{v}_{j'}^{l'})^T \mathbf{H}_{ji} \mathbf{u}_i^k]^T = 0, \quad (34)$$

for $j' \in \mathcal{I}_i^{\text{bs}} \cup \{j\}$, $1 \le l' \le \sum_{i' \in \mathcal{T}_{j'}} \sigma_{i'}$, and $(j', l') \neq (j, l)$.

(33) and (34) assure that each stream $\hat{s}_i^k$ ($i \in \mathcal{N}, 1 \le k \le \hat{\sigma}_i$) can be transported free of interference in the downlink. Therefore, we conclude that IA scheme $\hat{\pi}$ is feasible for the downlink. This completes the proof.

APPENDIX E
NETWORK THROUGHPUT OPTIMIZATION UNDER NO-IA SCHEME

In the no-IA scheme, a subset of subcarriers is allocated to each user for its data transmission such that at each BS, each data or interfering stream occupies a *unique* subcarrier. That is, there is a complete absence of overlapping of interfering streams on any subcarrier. Denote $\mathcal{K}$ as the set of subcarriers in the network. Denote w_{ik} as a binary variable to indicate whether the kth subcarrier is used by user i. Specifically, $w_{ik} = 1$ if the kth subcarrier is used for data transmission at user i and $w_{ik} = 0$ otherwise. Thus, the number of outgoing streams from user i can be expressed as

$$\sigma_i = \sum_{k \in \mathcal{K}} w_{ik}, \qquad i \in \mathcal{N}. \quad (35)$$

At BS $j \in \mathcal{M}$, a subcarrier $k \in \mathcal{K}$ can be used by only one user within its transmission range and interference range. Otherwise, it will cause interference collision or overlapping. Thus, we have the following constraints:

$$\sum_{i \in \mathcal{T}_j \cup \mathcal{I}_j} w_{ik} \le 1, \qquad k \in \mathcal{K}, j \in \mathcal{M}. \quad (36)$$

Therefore, the throughput maximization problem under no-IA scheme can be formulated as follows:

OPT-noIA:	Max	$r_{\min}$
	S.t.	(16), (35), (36);

where w_{ik} and σ_i are variables; while $\mathcal{C}_j^{\text{usr}}$, $\mathcal{O}_j^{\text{usr}}$, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{K}$ are known a priori based on the network topology and setting.

APPENDIX F
NETWORK THROUGHPUT OPTIMIZATION UNDER CRUDE-IA SCHEME

We formulate the same network throughput problem under the crude-IA scheme. In the crude-IA scheme, a subset of subcarriers is allocated to each user for its data transmission such that at each BS, each of its desired data streams is on a unique subcarrier while the interfering streams are allowed to overlap.

Recall that $\mathcal{K}$ is the set of subcarriers in the network and w_{ik} is a binary variable indicating whether the kth subcarrier is used at user i. Then, the number of outgoing streams from user i can be expressed as

$$\sigma_i = \sum_{k \in \mathcal{K}} w_{ik}, \qquad i \in \mathcal{N}. \quad (37)$$

Consider an BS $j \in \mathcal{M}$ and its serving users (i.e. users in $\mathcal{T}_j^{\text{usr}}$). To avoid transmission conflict, at most one of the users in $\mathcal{T}_j^{\text{usr}}$ can use the kth subcarrier for data stream transmission. Thus we have $\sum_{i \in \mathcal{T}_j^{\text{usr}}} w_{ik} \le 1$. If none of the users in $\mathcal{T}_j^{\text{usr}}$ uses the kth subcarrier for data stream transmission, then this subcarrier can accommodate any amount of interference (i.e., $\frac{1}{N} \sum_{i \in \mathcal{I}_j^{\text{usr}}} w_{ik} \le 1$). Combining these two cases, the interference avoidance scheme can be modeled by the following constraint:

$$\sum_{i \in \mathcal{T}_j^{\text{usr}}} w_{ik} + \frac{1}{N} \sum_{i \in \mathcal{I}_j^{\text{usr}}} w_{ik} \le 1, \qquad j \in \mathcal{M}, k \in \mathcal{K}. \quad (38)$$

Recall that $\mathcal{C}_j^{\text{usr}}$ is the set of users within the transmission range of BS j and $\mathcal{O}_j^{\text{usr}}$ is the set of users within the interference range of BS j. A user may be within the transmission range of multiple BSs and we use x_{ij} to indicate which BS is serving for it. Thus we have $\mathcal{T}_j^{\text{usr}} = \{i : i \in \mathcal{C}_j^{\text{usr}}, x_{ij} = 1\}$ and $\mathcal{I}_j^{\text{usr}} = \mathcal{O}_j^{\text{usr}} \cup \{i : i \in \mathcal{C}_j^{\text{usr}}, x_{ij} = 0\}$. Then the interference avoidance constraints in the network can be expressed as

$$\sum_{i \in \mathcal{C}_j^{\text{usr}}} x_{ij} w_{ik} + \frac{1}{N} \sum_{i \in \mathcal{C}_j^{\text{usr}}} (1 - x_{ij}) w_{ik} + \frac{1}{N} \sum_{i \in \mathcal{O}_j^{\text{usr}}} w_{ik} \le 1, \qquad j \in \mathcal{M}, k \quad (39)$$

To eliminate the nonlinear term $x_{ij} w_{ik}$ in (39), we define a new variable as follows:

$$q_{ijk} = x_{ij} w_{ik}, \qquad j \in \mathcal{M}, i \in \mathcal{C}_j^{\text{usr}}, k \in \mathcal{K}. \quad (40)$$

Given that both x_{ij} and w_{ik} are binary variables, it it easy to verify that constraint (40) is equivalent to the combination of the following three linear constraints:

$$q_{ijk} \le x_{ij}, \qquad i \in \mathcal{C}_j^{\text{usr}}, j \in \mathcal{M}, k \in \mathcal{K}. \quad (41)$$

$$q_{ijk} \le w_{ik}, \quad i \in \mathcal{C}_j^{\text{usr}}, j \in \mathcal{M}, k \in \mathcal{K}. \tag{42}$$

$$q_{ijk} \ge x_{ij} + w_{ik} - 1, \quad i \in \mathcal{C}_j^{\text{usr}}, j \in \mathcal{M}, k \in \mathcal{K}. \tag{43}$$

Replacing $x_{ij}w_{ik}$ by q_{ijk} in interference avoidance constraint (39), we have

$$\frac{N-1}{N} \sum_{i \in \mathcal{C}_j^{\text{usr}}} q_{ijk} + \frac{1}{N} \sum_{i \in \mathcal{C}_j^{\text{usr}} \cup \mathcal{O}_j^{\text{usr}}} w_{ik} \le 1, \quad j \in \mathcal{M}, k \in \mathcal{K}. \tag{44}$$

Therefore, the throughput maximization problem under crude-IA scheme can be formulated as follows:

OPT-crudeIA:	Max	$r_{\min}$
	S.t.	(9), (16), (44), (41), (42), (43), (37),

where x_{ij}, w_{ik}, σ_i, q_{ijk}, and $r_{\min}$ are variables; while $\mathcal{C}_j^{\text{usr}}$, $\mathcal{O}_j^{\text{usr}}$, N, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{K}$ are known a priori based on the network topology and setting.

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