

Nullification in the Air: Interference Neutralization in Multi-Hop Wireless Networks

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Abstract—Interference neutralization (IN) is an interference management technique that allows simultaneous transmission of multiple links by nullifying their mutual interference in the air via cooperation among the transmitters. Although IN has been studied from information theoretic perspective, its potential for a general multi-hop wireless network has not been explored. The goal of this paper is to understand IN in a multi-hop wireless network from networking perspective. We first establish an IN reference model. Based on this reference model, we develop a set of feasibility constraints for a subset of links to be active simultaneously. By identifying each eligible neutralization node (called *neut*), we study IN in a general multi-hop network and develop a set of necessary constraints to characterize neut selection, IN, and scheduling. These constraints allow us to study the performance of multi-hop networks without the need of getting involved into onerous signal design issues at the physical layer. Finally, we apply our IN model and constraints to study a throughput maximization problem and show that the use of IN can generally increase network throughput. In particular, throughput gain is most significant when the node density increases.

I. INTRODUCTION

Traditional paradigm for interference management in wireless networks is *avoidance*, where interference is avoided by orthogonalizing channel access, either in time, frequency, space, or code. Recent advancement of physical (PHY) layer technologies allows a shift in paradigm toward interference exploitation, which allows multiple concurrent independent transmissions in the interference domain. This is accomplished by the use of some powerful PHY layer techniques at a node's transceiver (e.g., interference cancellation (IC) and interference alignment (IA)). This paper studies *interference neutralization* (IN), which is a new technique in the interference exploitation family. The IN terminology was coined by Mohajer et al. in [9], [10], [11] when studying two-hop relay networks. As a special form of transmitter-side zero-forcing, IN refers to a joint design of signals at the transmitters so that these transmit signals nullify themselves in the air at their unintended receivers while remaining resolvable at their intended receivers. To achieve interference nullification in the air, IN requires that multiple transmitters have the same data that is under transmission. This makes it uniquely suitable for wireless mesh networks. Note that IN differs from transmitter-side IC, which projects interference to a perpendicular direction of the desired signal at the unintended receivers (rather than nullifying interference in the air). IN is also different from IA. Although both IN and IA require a joint signal design at the transmitters, the signal design of IN is to nullify interference in the air, while the signal design of IA is to align interference in the same direction.

To illustrate the idea of IN, let's consider three transmit nodes

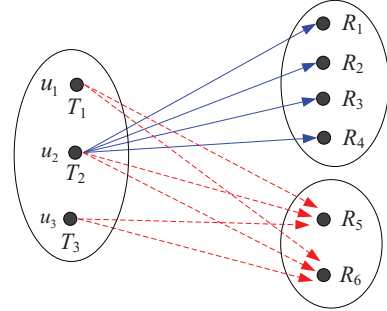


Fig. 1: An example that illustrates IN.

and six receive nodes as shown in Fig. 1. Each node only has a single antenna. Suppose that the three transmit nodes $\{T_1, T_2, T_3\}$ have the same data information x for transmission. Also suppose that $\{R_1, R_2, R_3, R_4\}$ are the intended receivers of x and $\{R_5, R_6\}$ are the unintended receivers. Further, we require that the interference on $\{R_5, R_6\}$ from $\{T_1, T_2, T_3\}$ be nullified. Denote h_{ji} as the channel coefficient between receive node j and transmit node i , which is a complex number. Denote u_i as the precoding coefficient at transmit node i , which is a complex number as well. Denote y_j as the received signal (desired signal or interference) at receive node j . Then we have

$$y_j = (h_{j1}u_1 + h_{j2}u_2 + h_{j3}u_3)x,$$

for receiver R_j , $j = 1, 2, \dots, 6$. We now show that through careful design of the precoding coefficients at the three transmitters $\{T_1, T_2, T_3\}$, the interference at the two unintended receive nodes $\{R_5, R_6\}$ can be neutralized while the desired signal at the four intended receivers $\{R_1, R_2, R_3, R_4\}$ can be decoded successfully. To do so, we need to show that there exist precoding coefficients (u_1 , u_2 , and u_3) that satisfy the following four constraints:

$$h_{j1}u_1 + h_{j2}u_2 + h_{j3}u_3 \neq 0, \quad j \in \{1, 2, 3, 4\}, \quad (1a)$$

$$h_{j1}u_1 + h_{j2}u_2 + h_{j3}u_3 = 0, \quad j \in \{5, 6\}. \quad (1b)$$

Suppose that the channel coefficients h_{ji} ($j = 1, 2, \dots, 6$ and $i = 1, 2, 3$) are independent of each other, it is not difficult to find a set of precoding coefficients u_1 , u_2 , u_3 to meet the six linear constraints. In fact, it can be shown that if one can find a set of nonzero value for u_1 , u_2 , and u_3 that satisfy (1b), then u_1 , u_2 , u_3 will satisfy (1a) almost surely [17]. Therefore, to find a feasible solution to (1), one only need to focus on (1b). For two equations with three variables, there exist non-unique

feasible solutions. For example, we can set $u_1 = 1$ and solve u_2 and u_3 in (1b). We have

$$u_2 = (h_{61}h_{53} - h_{63}h_{51}) / (h_{63}h_{52} - h_{62}h_{53}),$$

$$u_3 = (h_{61}h_{52} - h_{62}h_{51}) / (h_{62}h_{53} - h_{52}h_{63}).$$

It is easy to verify that the above precoding coefficients satisfy all constraints in (1). This indicates that by jointly constructing precoding coefficients at transmitters T_1 , T_2 , and T_3 , the interference at R_5 and R_6 can be nullified in the air while the signal at R_1 , R_2 , R_3 and R_4 can be successfully decoded. In this example, as nodes T_1 and T_2 help T_2 to cancel its interference at its unintended receive node, we call T_1 and T_2 as T_2 's neuts. In general, as we shall see in Section II, for K transmitters, we can neutralize their interference to $(K-1)$ unintended receivers while there is no limit on the number of intended receivers.

Conceptually, IN resembles distributed MIMO in the sense that both of them exploit precoding technique to achieve interference nullification at a selected subset of receivers. But there is a key difference between their objectives. Historically, the goal of distributed MIMO was mainly to maximize the SINR of users in a cellular network (rather than to nullify interference at a subset of receivers). Therefore, we choose to use terminology IN in this paper (as in [9], [10], [11]) so as to contrast its objective with that of distributed MIMO.

The next question is: why is IN particularly suitable for wireless mesh networks? We answer this question by applying IN to a wireless mesh network. Consider the wireless mesh network in Fig. 2 as an example. There are three sessions in the network: $S_1 \rightarrow D_1$, $S_2 \rightarrow D_2$, and $S_3 \rightarrow D_3$. Each session goes through multi-hop transmissions to deliver message from its source to its destination. Inside the network, due to broadcast advantage of wireless channel, there will be multiple nodes overhearing the same message from one of the source nodes and each of them can help relay the message to its next-hop nodes. In this regard, consider the scenario in Fig. 2, where the set of nodes in \mathcal{T} can send message to the set of nodes in \mathcal{R} in one hop. Within the set of nodes in \mathcal{T} , there are three subsets of nodes, differentiated by the legends \star , \blacksquare , and \blacktriangle , each of which has message from S_1 , S_2 , and S_3 , respectively. By using IN, the three subsets of nodes in \mathcal{T} can send their respective messages to their intended receivers in \mathcal{R} (marked with the three different legends) *simultaneously, in one time slot (instead of three)*. Inside \mathcal{R} , three subsets of nodes can successfully decode their own desired incoming messages since the undesired messages (interferences) can be neutralized by IN.

The above example illustrates the potential benefits of IN in a multi-hop network, i.e., allowing multiple data transmissions in the same interference domain. To date, the previous arts of IN have been limited to simple and specialized network configurations, such as $2 \times 2 \times 2$ network in [2], [15], half-duplex relay channel [1], [12], full-duplex relay channel [6], instantaneous relay channel in [3], [5], and ZZ network in [9], [10], [11]. It is not clear how IN can be systematically exploited in a general multi-hop network, such as the multi-hop network shown in Fig. 2. This knowledge gap underscores both the technical depth of this problem and the critical need of bridging this gap. The goal of this paper is to make a concrete step toward

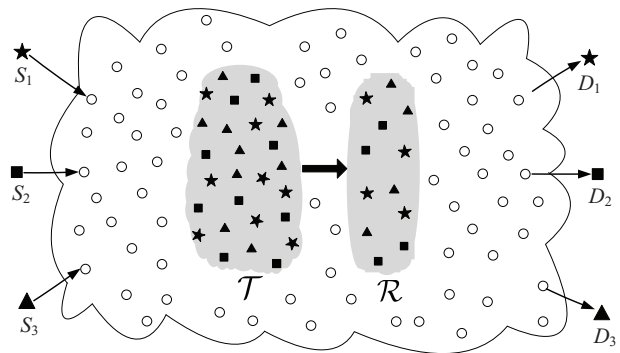


Fig. 2: An example that illustrates IN in a wireless mesh network.

advancing IN technique in a general multi-hop network. The main contributions of this paper are summarized as follows:

- We establish a reference model for IN. Under the reference model, we derive the maximum number of unintended receivers whose interference can be effectively neutralized. By applying the IN reference model to a set of links, we derive a set of feasibility constraints for a subset of links that can be active simultaneously.
- We show how IN can enable simultaneous data transmissions at different nodes in an ad hoc network by taking the broadcast advantage of wireless channels and the assistance of idle nodes. We introduce a concept called *neut* to represent those idle nodes that can be exploited for IN.
- Based on the notion of neut, we study IN in a general multi-hop network. We show that the thrust of the problem is to select an optimal set of neuts along each session's routing path. Subsequently, we develop the necessary mathematical models (constraints) to characterize neut selection, IN, and scheduling. Collectively, these constraints allow us to quantify the throughput benefits of IN at network level while not getting involved into the tedious signal design at the PHY layer.
- As an application, we apply our IN model to study a throughput maximization problem. We compare the throughput of the network with IN against that without IN. Simulation results show that the use of IN can generally increase throughput. We find that the throughput gain is most profound when the network is dense and the number of idle nodes that can be used as neuts is abundant.

The remainder of the paper is organized as follows. In Section II, we establish a reference model for IN, based on which we characterize the feasibility constraints for a set of active links. In Section III, we develop a mathematical model for IN in a multi-hop network. In Section IV, we apply our IN model to study a throughput maximization problem. Section V presents performance evaluation results. Section VI concludes this paper.

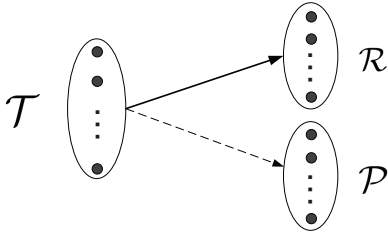


Fig. 3: A reference model for IN.

II. FEASIBILITY CONSTRAINTS FOR INTERFERING LINKS IN A SINGLE HOP

In this section, we study the feasibility conditions (constraints) for IN to work in a single-hop transmission. These constraints will lay the groundworks for our study of IN for multi-hop in the next section.

A Reference Model and Basic Result. Figure 3 shows the reference model (or building block) for IN in our study. Denote \mathcal{T} as the set of transmitters and \mathcal{R} as the set of intended receivers. Each receiver in \mathcal{R} is in the transmission range of the transmitters in \mathcal{T} . Denote \mathcal{P} as the set of receivers that are interfered with by all the transmitters in \mathcal{T} . That is, each receiver $j \in \mathcal{P}$ is within the interference range of every transmitter $i \in \mathcal{T}$. In the figure, the solid arrow line represents “aggregate” transmission link from the transmitters in \mathcal{T} to the receivers in \mathcal{R} . Such an aggregate link refers to concurrent transmissions from \mathcal{T} to \mathcal{R} . Likewise, the dashed arrow line represents “aggregate” interference link from transmitters in \mathcal{T} to unintended receivers in \mathcal{P} . Again, such an aggregate link refers to concurrent interference from \mathcal{T} to \mathcal{P} . In the following, if there is no ambiguity, we drop the wording “aggregate” when we refer to such links.

For the transmitters in \mathcal{T} , they may not be able to neutralize their interference to all the receivers in \mathcal{P} . So we ask the following question: How many receivers in \mathcal{P} can have their interference be neutralized by the transmitters in \mathcal{T} ?

Define \mathcal{Q} as a subset of \mathcal{P} (i.e., $\mathcal{Q} \subseteq \mathcal{P}$). To have the transmitters in \mathcal{T} neutralize their interference to each receiver in \mathcal{Q} while keeping their signals decodable at each receiver in \mathcal{R} , we need to ensure that the following linear constraints have a feasible solution:

$$\sum_{i \in \mathcal{T}} h_{ji} u_i \neq 0, \quad j \in \mathcal{R}; \quad (2)$$

$$\sum_{i \in \mathcal{T}} h_{ji} u_i = 0, \quad j \in \mathcal{Q}; \quad (3)$$

where h_{ji} is assumed to be a constant and u_i is a variable.

Since the channel coefficients are independent among themselves, it was shown in [17] that a *nonzero* solution to (3) also satisfies (2) almost surely (with probability 1). Therefore, we only need to make sure that there exists a *nonzero* solution to (3). For the linear equations in (3), there are $|\mathcal{Q}|$ constraints and $|\mathcal{T}|$ variables. According to [8, Ch. 2], a sufficient condition for the existence of nonzero solution to (3) is that it has more variables than constraints, i.e., $|\mathcal{Q}| \leq |\mathcal{T}| - 1$. Therefore, (2) and (3) always have a feasible solution if $|\mathcal{Q}| \leq |\mathcal{T}| - 1$. Simply

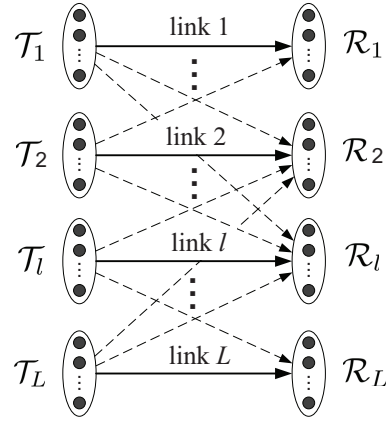


Fig. 4: A set of “aggregate” transmission links.

put, the transmitters in \mathcal{T} can neutralize their interference to $|\mathcal{T}| - 1$ receivers.

To find a set of precoding coefficients that satisfy (3), we can use Gauss–Jordan elimination algorithm. By performing row reduction on (3), we can obtain the reduced row echelon form as follows:

$$\begin{aligned} u_1 + \tilde{h}_{12}u_2 + \tilde{h}_{13}u_3 + \cdots + \tilde{h}_{1(T-1)}u_{(T-1)} + \tilde{h}_{1T}u_T &= 0, \\ u_2 + \tilde{h}_{23}u_3 + \cdots + \tilde{h}_{2(T-1)}u_{(T-1)} + \tilde{h}_{2T}u_T &= 0, \\ &\vdots \\ u_{(T-1)} + \tilde{h}_{QT}u_T &= 0, \end{aligned} \quad (4)$$

where $T = |\mathcal{T}|$ and $Q = |\mathcal{Q}|$.

In the reduced row echelon form (4), if a variable is a leading variable for an equation, then it is a basic variable; otherwise, it is a free variable [8, Ch. 2]. Denote \mathcal{G} as the set of basic variables and \mathcal{E} as the set of free variables. Then, a solution to (3) can be computed as follows:

$$u_i = \begin{cases} 1, & \text{for } u_i \in \mathcal{E}, \\ -\sum_{k=i+1}^T \tilde{h}_{ik}u_k, & \text{for } u_i \in \mathcal{G}. \end{cases} \quad (5)$$

Given that $|\mathcal{Q}| \leq |\mathcal{T}| - 1$, there exists at least one free variable in (4), i.e., $\mathcal{E} \neq \emptyset$. Since the free variable is set to 1 in (5), the solution is nonzero and thus satisfies (2) almost surely. The following lemma summarizes our results:

Lemma 1: *Through joint design of precoding coefficients, the transmitters in \mathcal{T} can neutralize their interference to any subset of receivers \mathcal{Q} in \mathcal{P} if $|\mathcal{Q}| \leq |\mathcal{T}| - 1$.*

Feasibility Constraints. We now apply the results in Lemma 1 to a general one-hop transmission scenario as shown in Fig. 4. In this figure, there is a set \mathcal{L} of concurrent links, where each link $l \in \mathcal{L}$ denotes aggregate transmissions from the transmitters in \mathcal{T}_l to the receivers in \mathcal{R}_l , as we discussed previously. For each transmitter in \mathcal{T}_l , we assume it has a fixed interference range. Then, for the receivers of link k (e.g., \mathcal{R}_k), some of them may be interfered with by the transmitters in \mathcal{T}_l and some of them may not. Denote \mathcal{P}_l as the set of receivers that are interfered with by at least one transmitter in \mathcal{T}_l . Then the subset

of receivers in \mathcal{R}_k that are interfered with by the transmitters in \mathcal{T}_l can be written as $\mathcal{P}_l \cap \mathcal{R}_k$.

Denote α_l as a binary variable to indicate whether link $l \in \mathcal{L}$ is active. Specifically, $\alpha_l = 1$ if link l is active (i.e., the transmitters in \mathcal{T}_l are transmitting to the receivers in \mathcal{R}_l) and 0 otherwise. Obviously, not all links in \mathcal{L} may be allowed to be active at the same time. So the question is: for a given subset of links, how can we determine if they can be active at the same time? In what follows, we present feasibility constraints that can be used to make this determination. Further, this feasibility constraint can be used to plot the entire feasibility region for $(\alpha_1, \alpha_2, \dots, \alpha_L)$.

To explore the feasibility constraints, we consider the following two cases for link $l \in \mathcal{L}$:

- *Link l is active (i.e., $\alpha_l = 1$).* In this case, the transmitters in \mathcal{T}_l must neutralize their interference to all receivers in \mathcal{P}_l that are receiving signals from their intended transmitters. To do so, the number of transmitters in \mathcal{T}_l should be at least 1 more than the number of active receivers in \mathcal{P}_l (based on Lemma 1). We now count the number of active receivers in \mathcal{P}_l . For link $k \in \mathcal{L}$, the subset of its intended receivers in \mathcal{R}_k that is also within \mathcal{P}_l is $\mathcal{P}_l \cap \mathcal{R}_k$. So the total number of active receivers in \mathcal{P}_l is $\sum_{k \neq l} |\mathcal{P}_l \cap \mathcal{R}_k| \cdot \alpha_k$. Based on Lemma 1, it is required that $\sum_{k \neq l} |\mathcal{P}_l \cap \mathcal{R}_k| \cdot \alpha_k \leq |\mathcal{T}_l| - 1$.
- *Link l is inactive (i.e., $\alpha_l = 0$).* In this case, the transmit nodes in \mathcal{T}_l do not generate interference. So there is no restriction on the number of active receivers within their interference ranges.

By defining N as the total number of nodes in the network, it is easy to verify that the above two cases can be combined as follows:

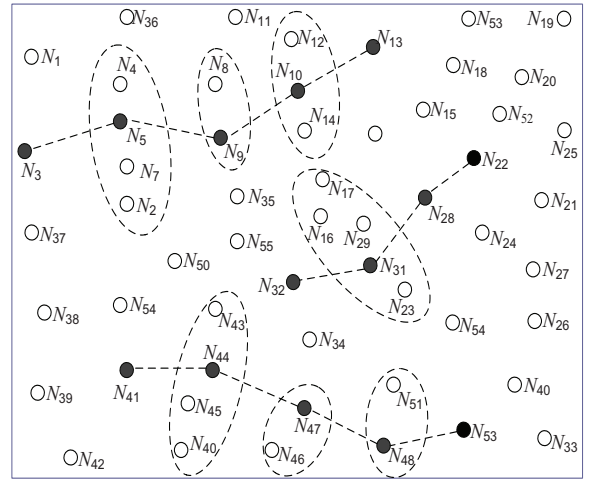
$$\sum_{k \neq l} |\mathcal{P}_l \cap \mathcal{R}_k| \cdot \alpha_k \leq |\mathcal{T}_l| - 1 + (1 - \alpha_l) \cdot N, \quad l \in \mathcal{L}. \quad (6)$$

Another restriction of neutralization in this network is that the transmitters in \mathcal{T}_l can only neutralize their interferences to a node that are interfered with by all transmitters in \mathcal{T}_l . To model this restriction, we define a binary constant for links l and k in Fig. 4 as follows:

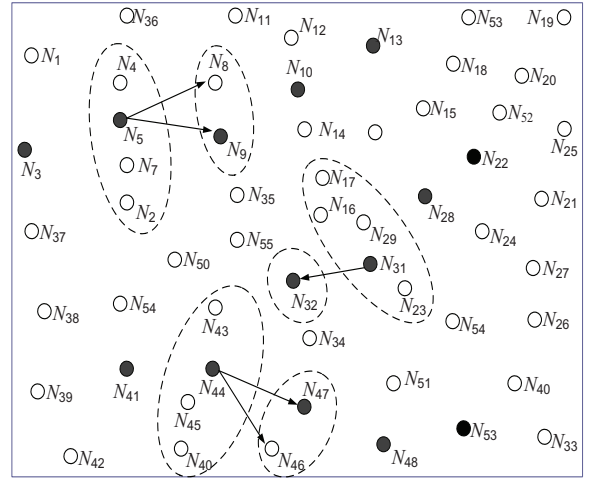
$$W_{l,k} = \begin{cases} 0 & \text{If there exists a receiver } j \in \mathcal{R}_k \text{ that is interfered with by a subset of transmitters in } \mathcal{T}_l, \\ 1 & \text{otherwise.} \end{cases}$$

Based on the definition of $W_{l,k}$, we have the following two cases:

- $W_{l,k} = 0$: In this case, receiver $j \in \mathcal{R}_k$ is interfered with by some transmitter(s) in \mathcal{T}_l , but not interfered with by all the transmitters in \mathcal{T}_l . In our IN scheme, to guarantee the feasibility of the final solution, neutralization is only done for a receiver that is interfered with by all the transmitters in \mathcal{T}_l . So we will not employ IN to nullify the interference for receiver $j \in \mathcal{R}_k$. As a result, links k and l cannot be active at the same time, i.e., $\alpha_l + \alpha_k \leq 1$.
- $W_{l,k} = 1$: In this case, there is no additional restriction on α_l and α_k .



(a) A network with three sessions. Each oval contains a node along a session's path and a set of nodes acting as neutrals.



(b) IN among three active links in a time slot.

Fig. 5: An example of IN in a multi-hop wireless network.

It is easy to verify that the above two cases can be combined into the following constraint:

$$\alpha_l + \alpha_k \leq 1 + W_{l,k}, \quad l, k \in \mathcal{L}, l \neq k. \quad (7)$$

Therefore, (6) and (7) constitute the feasibility constraints for $(\alpha_1, \alpha_2, \dots, \alpha_L)$. By enumerating all possible setting of $(\alpha_1, \alpha_2, \dots, \alpha_L)$, we can use (6) and (7) to plot the feasible region.

III. INTERFERENCE NEUTRALIZATION IN MULTI-HOP NETWORKS

In this section, we study IN in multi-hop networks. Consider a multi-hop network consisting of a set of nodes as shown in Fig. 5(a). Among the nodes, there is a set of sessions \mathcal{S} , with $\text{src}(s)$ and $\text{dst}(s)$ denoting the source and destination nodes of session $s \in \mathcal{S}$, respectively. Denote $r(s)$ as the end-to-end data rate of session $s \in \mathcal{S}$. We assume that the routing path of each session is obtained through some routing protocol for ad hoc networks. Based on the routing paths, the nodes in the network can be classified into two subsets: \mathcal{N}_s and $\mathcal{N}_{\text{idle}}$, where \mathcal{N}_s is

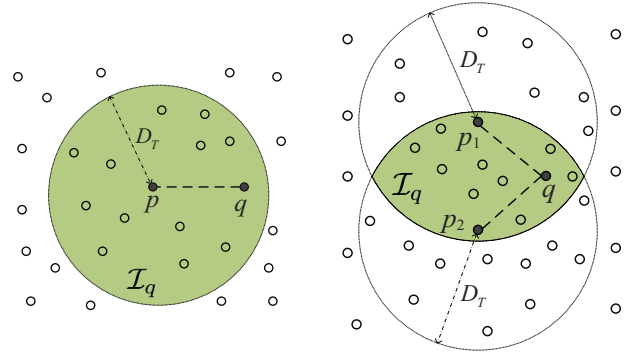
the set of nodes on routing paths (marked as filled circles in Fig. 5(a)), and $\mathcal{N}_{\text{idle}}$ is the set of remaining nodes (marked as empty circles). Denote \mathcal{L}_s as the set of links that are traversed by the set of sessions \mathcal{S} . Assume that transmission scheduling among the links in \mathcal{L}_s is based on a frame that consists of T time slots.

For each multi-hop session, if an idle node is chosen to help a session node cancel its interference at its unintended nodes, then we call this idle node *neut*. Figure 5(a) illustrates how some idle nodes along the path are chosen as neuts by the nodes along each path. For example, $\{N_2, N_4, N_7\}$ are chosen as N_5 's neuts, $\{N_8\}$ is chosen as N_9 's neut, $\{N_{12}, N_{14}\}$ are chosen as N_{10} 's neuts, and so forth. When receiving, the group of neuts can receive the same information as the corresponding node on the path from its one-hop upstream node. For example, $\{N_2, N_4, N_7\}$, which are N_5 's neuts, can receive data from N_3 since they are all within the transmission range of N_3 . When transmitting, based on Lemma 1, $\{N_2, N_4, N_5, N_7\}$ can neutralize their interferences to three other unintended receivers via the joint design of their precoding coefficients. Fig. 5(b) shows an example where IN can enable simultaneous transmissions in a multi-hop wireless network. Suppose all nodes are in the same interference domain. $\{N_2, N_4, N_5, N_7\}$ can neutralize their interferences to receive nodes N_{32} , N_{46} , and N_{47} . Similarly, the set of transmitters $\{N_{16}, N_{17}, N_{23}, N_{29}, N_{31}\}$ can neutralize their interferences to receive nodes N_8 , N_9 , N_{46} , and N_{47} ; the set of transmitters $\{N_{40}, N_{43}, N_{44}, N_{45}\}$ can neutralize their interference to receive nodes N_8 , N_9 , and N_{32} . Since interference is neutralized at the receivers, the three transmissions can be active in the same time slot. Note that without IN, only one of the three links ($N_5 \rightarrow N_9$, $N_{44} \rightarrow N_{47}$, or $N_{31} \rightarrow N_{32}$) can be active in a time slot.

It is worth pointing out that for a node $q \in \mathcal{N}_s$, how to choose a subset of its neighboring nodes as neuts is a key problem in IN. A small number of neuts will limit the neutralization capability (see Lemma 1) while a large number of neuts may increase the interference footprint unnecessarily. For the same number of neuts, the locations of the neuts are also important as they will characterize the overall shape and size of the interference footprint. Therefore, one must choose a set of neuts for each node $q \in \mathcal{N}_s$ meticulously and systematically to maximize the benefits of IN. In our study for IN in a multi-hop network, we model neut selection as part of the optimization problem.

A. Neut Selection and IN

Referring to Fig. 5(a), for a node on a path, say N_5 , not every idle node is eligible to be its neut. An idle node is eligible to be N_5 's neut only if it can receive the same information from N_5 's one-hop upstream node, i.e., N_3 . In general, for a node $q \in \mathcal{N}_s$, denote \mathcal{I}_q as the set of idle nodes that are eligible to be its neuts. Referring to Fig. 6(a), if node q has only one one-hop upstream node, say p , then \mathcal{I}_q is the set of idle nodes within the transmission range of node p , i.e., $\mathcal{I}_q = \{i : d(i, p) \leq D_T, i \in \mathcal{N}_{\text{idle}}\}$, where $d(i, p)$ is the distance between nodes i and p and D_T is a node's transmission range. If node $q \in \mathcal{N}_s$ is traversed by multiple sessions (see, e.g., Fig. 6(b)), node q has multiple one-hop upstream nodes and therefore \mathcal{I}_q is the



(a) The case when q is traversed by only one session (b) The case when q is traversed by multiple sessions

Fig. 6: An illustration of \mathcal{A}_q – the set containing eligible neuts for a node q .

set of idle nodes that fall in the intersection of the transmission ranges of these nodes. Therefore, we have

$$\mathcal{I}_q = \bigcap_{(p,q) \in \mathcal{L}_s} \left\{ i : d(i, p) \leq D_T, i \in \mathcal{N}_{\text{idle}} \right\}, \quad q \in \mathcal{N}_s,$$

where (p, q) represents a one-hop link in \mathcal{L}_s .

Consider an idle node $i \in \mathcal{N}_{\text{idle}}$. It may be eligible to serve as a neut for multiple nodes in \mathcal{N}_s , but it can only serve as a neut for only one node in \mathcal{N}_s . Denote \mathcal{M}_i as a subset of nodes in \mathcal{N}_s for which idle node i is eligible to serve as a neut. Denote $\lambda_{q,i}$ as a binary variable to indicate whether or not node i is assigned to serve as a neut for node $q \in \mathcal{M}_i$. Specifically,

$$\lambda_{i \rightarrow q} = \begin{cases} 1 & \text{if node } i \text{ is assigned to be node } q \text{'s neut,} \\ 0 & \text{otherwise.} \end{cases}$$

Since an idle node can serve as a neut for at most one node, we have the following constraint:

$$\sum_{q \in \mathcal{M}_i} \lambda_{i \rightarrow q} \leq 1, \quad i \in \mathcal{N}_{\text{idle}}. \quad (8)$$

For notational convenience, we define \mathcal{A}_q as the union of \mathcal{I}_q and $\{q\}$, i.e.,

$$\mathcal{A}_q = \mathcal{I}_q \cup \{q\}. \quad (9)$$

We also define \mathcal{B}_q as the subset of idle nodes in \mathcal{A}_q that are chosen to be node q 's neuts, plus node q itself, i.e.,

$$\mathcal{B}_q = \{i : i \in \mathcal{A}_q, \lambda_{i \rightarrow q} = 1\}, \quad (10)$$

where $\lambda_{q \rightarrow q} \equiv 1$ for $q \in \mathcal{N}_s$.

For transmission scheduling among the links in \mathcal{L}_s , we assume it is based on time slots. Consider all links in \mathcal{L}_s in a time slot t as shown in Fig. 7. Obviously, not all these links can be active at the same time. Denote $\alpha_l(t)$ as a binary variable to indicate whether or not link $l \in \mathcal{L}_s$ is active in time slot t , i.e.,

$$\alpha_l(t) = \begin{cases} 1 & \text{if link } l \text{ is active in time slot } t, \\ 0 & \text{otherwise.} \end{cases}$$

Consider link l 's transmitters in $\mathcal{B}_{\text{Tx}(l)}$ and a node j among another link k 's receivers in $\mathcal{A}_{\text{Rx}(l)}$. We define a binary variable $\pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$ to indicate whether or not the transmitters in $\mathcal{B}_{\text{Tx}(l)}$

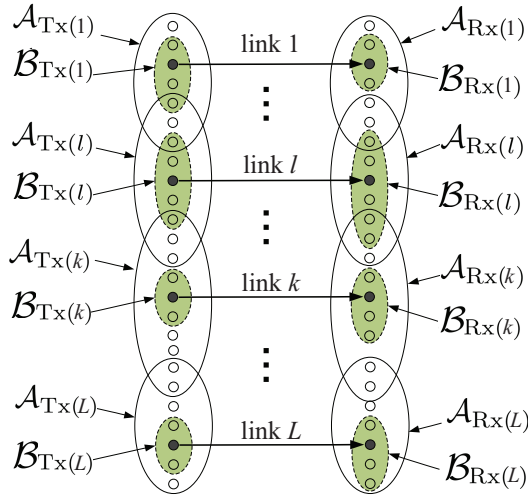


Fig. 7: IN constraints and link scheduling constraints. Interference between transmitters and receivers is not shown in the figure.

neutralize their interferences to node $j \in \mathcal{A}_{\text{Rx}(k)}$ in time slot t . That is,

$$\pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t) = \begin{cases} 1 & \text{if the transmitters in } \mathcal{B}_{\text{Tx}(l)} \text{ neutralize} \\ & \text{their interference to node } j \in \mathcal{A}_{\text{Rx}(k)} \\ & \text{in time slot } t, \\ 0 & \text{otherwise,} \end{cases}$$

Then, we have two cases:

- $\alpha_l(t) = 1$: In this case, the nodes in $\mathcal{B}_{\text{Tx}(l)}$ are active transmitters. Based on Lemma 1, we know that the transmitters in $\mathcal{B}_{\text{Tx}(l)}$ can neutralize their interferences to at most $(|\mathcal{B}_{\text{Tx}(l)}| - 1)$ unintended receivers. Based on (10), we have $|\mathcal{B}_{\text{Tx}(l)}| - 1 = (\sum_{i \in \mathcal{I}_{\text{Tx}(l)}} \lambda_{i \rightarrow \text{Tx}(l)} + 1) - 1 = \sum_{i \in \mathcal{I}_{\text{Tx}(l)}} \lambda_{i \rightarrow \text{Tx}(l)}$. Therefore, we have the following constraint:

$$\sum_{j \in \bigcup_{k \in \mathcal{L}_s, k \neq l} \mathcal{A}_{\text{Rx}(k)}} \pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t) \leq |\mathcal{B}_{\text{Tx}(l)}| - 1 = \sum_{i \in \mathcal{I}_{\text{Tx}(l)}} \lambda_{i \rightarrow \text{Tx}(l)}.$$

- $\alpha_l(t) = 0$: In this case, the nodes in $\mathcal{B}_{\text{Tx}(l)}$ are inactive. They neither produce any interference to other nodes nor possess IN capability. Therefore, we have $\pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t) = 0$ for all $j \in \mathcal{A}_{\text{Rx}(k)}$, $k \in \mathcal{L}_s \setminus \{l\}$.

It is easy to verify that the above two cases can be combined into the following equivalent set of constraints:

$$\sum_{j \in \bigcup_{k \in \mathcal{L}_s, k \neq l} \mathcal{A}_{\text{Rx}(k)}} \pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t) \leq \left(\sum_{i \in \mathcal{I}_{\text{Tx}(l)}} \lambda_{i \rightarrow \text{Tx}(l)} \right) \cdot \alpha_l(t), \quad l \in \mathcal{L}_s, 1 \leq t \leq T. \quad (11)$$

For (11), we implicitly assume that node $j \in \mathcal{A}_{\text{Rx}(k)}$ falls in the intersection of interference ranges of all nodes in $\mathcal{B}_{\text{Tx}(l)}$ when $\pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t) = 1$. We now develop the necessary constraints to ensure that this assumption is valid. Define a binary indicator $E_{i,j}$ to indicate whether or not node j is within node i 's interference range. That is,

$$E_{i,j} = \begin{cases} 1 & \text{if node } j \text{ is within node } i \text{'s interference range,} \\ 0 & \text{otherwise.} \end{cases}$$

If $\pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t) = 1$, then node j must be within the interference range of every node in $\mathcal{B}_{\text{Tx}(l)}$, i.e., $E_{i,j} = 1$ for $i \in \mathcal{B}_{\text{Tx}(l)}$. Given that $\lambda_{i \rightarrow \text{Tx}(l)} = 1$ for $i \in \mathcal{B}_{\text{Tx}(l)}$ and $\lambda_{i \rightarrow \text{Tx}(l)} = 0$ for $i \in \mathcal{A}_{\text{Tx}(l)} \setminus \mathcal{B}_{\text{Tx}(l)}$, it is required that $\lambda_{i \rightarrow \text{Tx}(l)} \leq E_{i,j}$ for $i \in \mathcal{A}_{\text{Tx}(l)}$. Otherwise (i.e., $\pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t) = 0$), there is no interference from $\mathcal{B}_{\text{Tx}(l)}$ and therefore there is no requirement on whether node j is within the interference range of the nodes in $\mathcal{B}_{\text{Tx}(l)}$. Combining these two cases, we have

$$\pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t) \cdot \lambda_{i \rightarrow \text{Tx}(l)} \leq \pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t) \cdot E_{i,j}, \quad l \in \mathcal{L}_s, i \in \mathcal{A}_{\text{Tx}(l)}, k \in \mathcal{L}_s \setminus \{l\}, j \in \mathcal{A}_{\text{Rx}(k)}, 1 \leq t \leq T. \quad (12)$$

B. Link Scheduling Constraints

For each node, we assume that it has a single antenna and operates with a half-duplex radio. When acting as a transmitter (or receiver), we assume a node can only be used by at most one active link in a time slot. Then we have

$$\sum_{q \in \{\text{Tx}(l), \text{Rx}(l)\}} \alpha_q(t) \leq 1, \quad q \in \mathcal{N}_s, 1 \leq t \leq T. \quad (13)$$

Consider node $j \in \mathcal{A}_{\text{Rx}(k)}$ in Fig. 7. We define a binary variable $\theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$ to indicate whether or not node j is within the interference range of at least one active transmitter in $\mathcal{B}_{\text{Tx}(l)}$. That is,

$$\theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t) = \begin{cases} 1 & \text{if node } j \text{ is within the interference range} \\ & \text{of at least one active transmitter in} \\ & \mathcal{B}_{\text{Tx}(l)} \text{ in time slot } t, \\ 0 & \text{otherwise.} \end{cases}$$

We now explore the relationship between $\theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$ and $\alpha_l(t) = 1$ as follows. If $\alpha_l(t) = 1$, then the nodes in $\mathcal{B}_{\text{Tx}(l)}$ are active transmitters. Based on the definition of $\theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$, $\theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t) = 0$ if and only if node j is out of the interference range of every node $i \in \mathcal{B}_{\text{Tx}(l)}$, i.e., $E_{i,j} = 0$ for $i \in \mathcal{B}_{\text{Tx}(l)}$. Therefore, we have

$$\frac{1}{N} \sum_{i \in \mathcal{B}_{\text{Tx}(l)}} E_{i,j} \leq \theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t) \leq \sum_{i \in \mathcal{B}_{\text{Tx}(l)}} E_{i,j},$$

where N is the total number of nodes in the network. Otherwise (i.e., $\alpha_l(t) = 0$), the nodes in $\mathcal{B}_{\text{Tx}(l)}$ are inactive. So they do not interfere with node j , i.e., $\theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t) = 0$. Combining these two cases, we have

$$\left[\frac{1}{N} \sum_{i \in \mathcal{B}_{\text{Tx}(l)}} E_{i,j} \right] \cdot \alpha_l(t) \leq \theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t) \leq \left[\sum_{i \in \mathcal{B}_{\text{Tx}(l)}} E_{i,j} \right] \cdot \alpha_l(t), \quad k \in \mathcal{L}_s, j \in \mathcal{A}_{\text{Rx}(k)}, l \in \mathcal{L}_s \setminus \{k\}, 1 \leq t \leq T. \quad (14)$$

Consider a receive node $j \in \mathcal{B}_{\text{Rx}(k)}$, $k \in \mathcal{L}_s$, in Fig. 7. Since it has a single antenna, it does not have capability to cancel interference while receiving. If node j is active and being interfered with by at least one node in $\mathcal{B}_{\text{Tx}(l)}$, then $\mathcal{B}_{\text{Tx}(l)}$ must neutralize such interference to node j . To meet this requirement, we have the following constraints for $\theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$ and $\pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$ in two cases:

- $\alpha_k(t) = 1$: In this case, link k is active and thus node j is an active receiver. If $\theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t) = 1$, node j is interfered with by $\mathcal{B}_{\text{Tx}(l)}$. To neutralize such interference to node j , we must have $\pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t) = 1$. Otherwise (i.e., $\theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t) = 0$), node j is not interfered with by $\mathcal{B}_{\text{Tx}(l)}$. Then $\mathcal{B}_{\text{Tx}(l)}$ do not need to neutralize their interferences to node j , i.e., $\pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t) = 0$. Combining these two cases, we have $\theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t) = \pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$.
- $\alpha_k(t) = 0$: In this case, link k is an inactive link and thus node j is an inactive receiver. Then node j can be interfered with by any unintended transmitters, i.e., $\theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$ can be either 0 or 1. Further, since node j is inactive, there is no requirement for $\mathcal{B}_{\text{Tx}(l)}$ to neutralize their interference to node j , i.e., $\pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$ can be either 0 or 1.

Combining these two cases, we have the following constraint:

$$-[1 - \alpha_k(t)] \leq \theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t) - \pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t) \leq [1 - \alpha_k(t)],$$

$$k \in \mathcal{L}_s, j \in \mathcal{B}_{\text{Rx}(k)}, l \in \mathcal{L}_s \setminus \{k\}, 1 \leq t \leq T. \quad (15)$$

Collectively, constraints (8), (11), (12), (13), (14), and (15) constitute an IN model, which allows us to determine which subsets of links can be active simultaneously without the need of involving onerous signal design at the PHY layer.

IV. AN APPLICATION TO THROUGHPUT MAXIMIZATION PROBLEM IN MULTI-HOP NETWORK

In this section, we apply IN to study a throughput maximization problem in a multi-hop network. The objective is to maximize the minimum achievable data rate among all sessions. To formulate this throughput maximization problem, let us consider a specific link $l \in \mathcal{L}_s$ in the network. It may be traversed by multiple sessions' routing paths. Denote \mathcal{S}_l as the set of sessions that share wireless link l along their paths. Then the aggregate data rate requirement on link l is $\sum_{s \in \mathcal{S}_l} r(s)$. For link l , we assume that fixed modulation and coding scheme (MCS) is used for its data stream transmission and one data stream in one time slot corresponds to one unit data rate. Then, the achievable data rate of link l (averaged over T time slots) is $\frac{1}{T} \sum_{t=1}^T \alpha_l(t)$. Since the aggregate data rate requirement (for its traversing sessions) cannot exceed the achievable data rate, we have

$$\sum_{s \in \mathcal{S}_l} r(s) \leq \frac{1}{T} \sum_{t=1}^T \alpha_l(t), \quad l \in \mathcal{L}. \quad (16)$$

By denoting r_{\min} as the minimum achievable rate among all sessions, we have

$$r_{\min} \leq r(s), \quad s \in \mathcal{S}. \quad (17)$$

Collectively, the throughput maximization problem can be formulated as follows:

OPT-IN^{raw}: max r_{\min}
s.t. Neut selection and IN: (8), (11), (12);
Link scheduling: (13), (14), (15);
Link rate constraints: (16);
Min rate constraints: (17).

In this optimization formulation, (11), (12), (14), and (15) are nonlinear constraints and the rest are all linear. We now show how to linearize (11), (12), (14), and (15) without loss of optimality.

Reformation of (11). To linearize (11), we define a new binary variable $\gamma_{i \rightarrow \text{Tx}(l)}(t) = \lambda_{i \rightarrow \text{Tx}(l)} \cdot \alpha_l(t)$. Then, (11) can be equivalently transformed to the following linear constraint:

$$\sum_{j \in \cup_{k \in \mathcal{L}_s}^{k \neq l} \mathcal{A}_{\text{Rx}(k)}} \pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t) \leq \sum_{i \in \mathcal{I}_{\text{Tx}(l)}} \gamma_{i \rightarrow \text{Tx}(l)}(t),$$

$$l \in \mathcal{L}_s, 1 \leq t \leq T. \quad (18)$$

Based on the definition of $\gamma_{i \rightarrow \text{Tx}(l)}(t)$, we can enumerate all possible cases for $\lambda_{i \rightarrow \text{Tx}(l)}$ and $\alpha_l(t)$ (both are binary variables) and obtain the following linear constraint for $\lambda_{i \rightarrow \text{Tx}(l)}$:

$$\lambda_{i \rightarrow \text{Tx}(l)} + \alpha_l(t) - 1 \leq \gamma_{i \rightarrow \text{Tx}(l)}(t) = \frac{1}{2}[\lambda_{i \rightarrow \text{Tx}(l)} + \alpha_l(t)],$$

$$l \in \mathcal{L}, i \in \mathcal{A}_{\text{Tx}(l)}, 1 \leq t \leq T. \quad (19)$$

Reformation of (12). In (12), $E_{i,j}$ is determined by the network topology and therefore it is a constant. But we have a product of two binary variables $\lambda_{i \rightarrow \text{Tx}(l)}$ and $\pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$. By enumerating all possibilities for these two binary variables, it is easy to verify that (12) is equivalent to the following linear constraint:

$$\pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t) + \lambda_{i \rightarrow \text{Tx}(l)} \leq 1 + E_{i,j},$$

$$l \in \mathcal{L}_s, i \in \mathcal{A}_{\text{Tx}(l)}, j \in \cup_{k \in \mathcal{L}_s}^{k \neq l} \mathcal{A}_{\text{Rx}(k)}, 1 \leq t \leq T. \quad (20)$$

Reformation of (14). In (14), $E_{i,j}$ is a constant but $\mathcal{B}_{\text{Tx}(l)}$ is a variable set. To linearize this constraint, we classify the nodes in $\mathcal{A}_{\text{Tx}(l)}$ into two subsets: $\mathcal{B}_{\text{Tx}(l)}$ and $\mathcal{A}_{\text{Tx}(l)} \setminus \mathcal{B}_{\text{Tx}(l)}$. For node $i \in \mathcal{B}_{\text{Tx}(l)}$, $\lambda_{i \rightarrow \text{Tx}(l)} = 1$. For node $i \in \mathcal{A}_{\text{Tx}(l)} \setminus \mathcal{B}_{\text{Tx}(l)}$, $\lambda_{i \rightarrow \text{Tx}(l)} = 0$. Then we have

$$\sum_{i \in \mathcal{B}_{\text{Tx}(l)}} E_{i,j} = \sum_{i \in \mathcal{A}_{\text{Tx}(l)}} E_{i,j} \cdot \lambda_{i \rightarrow \text{Tx}(l)}.$$

Based on the above result, constraint (14) can be equivalently written as:

$$\left[\frac{1}{N} \sum_{i \in \mathcal{A}_{\text{Tx}(l)}} E_{i,j} \cdot \lambda_{i \rightarrow \text{Tx}(l)} \right] \cdot \alpha_l(t) \leq \theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$$

$$\leq \left[\sum_{i \in \mathcal{A}_{\text{Tx}(l)}} E_{i,j} \cdot \lambda_{i \rightarrow \text{Tx}(l)} \right] \cdot \alpha_l(t),$$

$$k \in \mathcal{L}_s, j \in \mathcal{A}_{\text{Rx}(k)}, l \in \mathcal{L}_s \setminus \{k\}, 1 \leq t \leq T. \quad (21)$$

Constraint (21) is still nonlinear as it involves product of variables $\lambda_{i \rightarrow \text{Tx}(l)} \cdot \alpha_l(t)$. Based on our previous definition of $\gamma_{i \rightarrow \text{Tx}(l)}(t) = \lambda_{i \rightarrow \text{Tx}(l)} \cdot \alpha_l(t)$, nonlinear constraint (21) can be further linearized as

$$\frac{1}{N} \sum_{i \in \mathcal{A}_{\text{Tx}(l)}} E_{i,j} \cdot \gamma_{i \rightarrow \text{Tx}(l)}(t) \leq \theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$$

$$\leq \sum_{i \in \mathcal{A}_{\text{Tx}(l)}} E_{i,j} \cdot \gamma_{i \rightarrow \text{Tx}(l)}(t),$$

$$k \in \mathcal{L}_s, j \in \mathcal{A}_{\text{Rx}(k)}, l \in \mathcal{L}_s \setminus \{k\}, 1 \leq t \leq T. \quad (22)$$

Reformation of (15). For (15), it appears linear, but it is actually not. This is because the set of nodes in $\mathcal{B}_{\text{Rx}(k)}$ is unknown, which is determined by variable $\lambda_{j \rightarrow \text{Rx}(k)}$. To address this problem, we want to extend the range from $\mathcal{B}_{\text{Rx}(k)}$ to $\mathcal{A}_{\text{Rx}(k)}$ (a constant set) without any change to the solution space. Recall that $\lambda_{j \rightarrow \text{Rx}(k)} = 1$ for $j \in \mathcal{B}_{\text{Rx}(k)}$ and $\lambda_{j \rightarrow \text{Rx}(k)} = 0$ for $j \in \mathcal{A}_{\text{Rx}(k)} \setminus \mathcal{B}_{\text{Rx}(k)}$. Therefore, for $j \in \mathcal{A}_{\text{Rx}(k)}$, we have the following two cases:

- $\lambda_{j \rightarrow \text{Rx}(k)} = 1$: In this case, node j must be in $\mathcal{B}_{\text{Rx}(k)}$. Then node j must satisfy (15).
- $\lambda_{j \rightarrow \text{Rx}(k)} = 0$: In this case, node j must be in $\mathcal{A}_{\text{Rx}(k)} \setminus \mathcal{B}_{\text{Rx}(k)}$. Then node j does not need to satisfy (15).

Combining these two cases, $j \in \mathcal{B}_{\text{Rx}(k)}$ in (15) can be expanded to $j \in \mathcal{A}_{\text{Rx}(k)}$ by rewriting (15) as follows:

$$\begin{aligned}
 -[2 - \alpha_k(t) - \lambda_{j \rightarrow \text{Rx}(k)}] &\leq \theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t) - \pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t) \\
 &\leq [2 - \alpha_k(t) - \lambda_{j \rightarrow \text{Rx}(k)}], \\
 k &\in \mathcal{L}_s, j \in \mathcal{A}_{\text{Rx}(k)}, l \in \mathcal{L}_s \setminus \{k\}, 1 \leq t \leq T.
 \end{aligned} \tag{23}$$

Note that when $\lambda_{j \rightarrow \text{Rx}(k)} = 1$, (23) is exactly (15); when $\lambda_{j \rightarrow \text{Rx}(k)} = 0$, (23) does not impose any additional constraint on $\theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$, $\pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$, and $\alpha_k(t)$.

Problem Reformulation. In summary, by replacing nonlinear constraints (11), (12), (14), and (15) with (18), (19), (20), (22), and (23), we can reformulate OPT-IN^{raw} as follows:

OPT-IN: max r_{\min}
s.t. Neut selection and IN: (8), (18), (19), (20);
Link scheduling: (13), (22), (23);
Link rate constraints: (16);
Min rate constraints: (17);

where N , T , and $E_{i,j}$ are constant; \mathcal{N}_s , $\mathcal{N}_{\text{idle}}$, \mathcal{L}_s , \mathcal{S} , $\mathcal{A}_{\text{Tx}(l)}$, and $\mathcal{A}_{\text{Rx}(l)}$ are given sets; $\alpha_l(t)$, $\lambda_{i \rightarrow q}$, $\pi_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$, $\theta_j^{\mathcal{B}_{\text{Tx}(l)}}(t)$, and $\gamma_{i \rightarrow \text{Tx}(l)}(t)$ are optimization binary variables; r_{\min} and $r(s)$ are optimization continuous variables. Note that the reformulation does not change the optimality of the problem, i.e., OPT-IN^{raw} and OPT-IN have the same optimal objective value.

OPT-IN offers a centralized approach to exploit IN for throughput maximization in multi-hop networks. Some discussions of its solution in a practical network are in order. First, the transmitters in \mathcal{B}_q , $q \in \mathcal{N}_s$, are required to have cooperation when designing their precoding coefficients for IN. This is a mild requirement, as the nodes in \mathcal{B}_q are typically close to each other in a local area. Second, the transmitters in \mathcal{B}_q are required to have channel state information (CSI) between themselves and their interfering receivers. To send CSI from the receivers to the transmitters, strategies such as adaptive feedback compression (AFC) in [16] can be employed to reduce the overhead in feedback. Third, the transmitters in \mathcal{B}_q are required to be synchronized when transmitting their signals. This is also a mild requirement, as synchronization at the time slot level is sufficient. Synchronization protocols such as FTSP [7] can meet this requirement.

OPT-IN is in the form of mixed-integer linear program (MILP). Although a general MILP problem is NP-hard [13], there exist efficient optimal algorithms (e.g., branch-and-bound

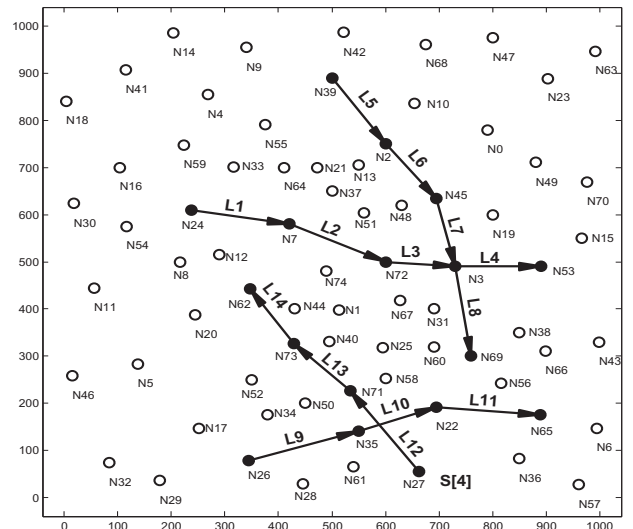


Fig. 8: A case study of IN in a multi-hop network.

with cutting planes [4, Ch. 5]) and efficient heuristic algorithms (e.g., sequential fixing [4, Ch. 10]) to solve it. For networks with a few hundred nodes, an off-the-shelf solver such as CPLEX [18] can be used. Since our goal is to study the performance gain of IN in multi-hop networks, we employ CPLEX in our performance evaluation.

V. PERFORMANCE EVALUATION

In this section, we first explain our simulation setting and then use a case study to show the throughput gain of IN in a network instance. Finally, we present our simulation results from a large number of network instances to show how IN affects the throughput performance of multi-hop networks. As a benchmark for performance comparison, we formulate the same throughput maximization problem when IN is not used, and denote it as OPT-noIN. OPT-noIN follows interference avoidance paradigm and schedules links in such a manner that there is no interference among the active links in a time slot.

A. Simulation Setting

Without loss of generality, we normalize the units of distance, data rate, bandwidth, and time with appropriate dimensions. We consider a multi-hop network consisting of a set of randomly generated nodes that are uniformly deployed in an 1000×1000 square region. Each node is equipped with a single antenna. The transmission and interference ranges are set to 250 and 500, respectively [14]. There are four sessions in each network instance, with their source and destination nodes being randomly chosen among all the nodes. The routing path of each session is computed based on a node's transmission range and the shortest path routing algorithm. We assume each source node has persistent traffic for transmission. We also assume that a frame has $T = 15$ time slots.

B. A Case Study

Figure 8 shows a network instance that consists of 75 nodes. Among the nodes, there are 4 sessions, with their source and destination nodes as well as their routing paths shown in

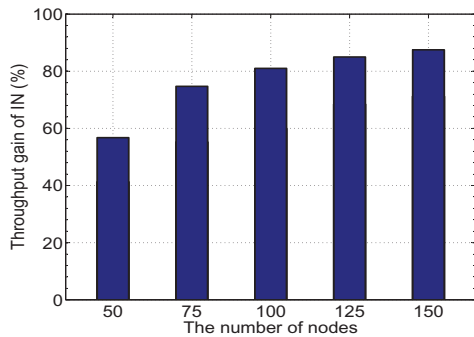


Fig. 9: Impact of node density.

the figure. By solving OPT-IN for this network instance, we obtained the optimal objective value of $2/15$, meaning that each session’s achievable end-to-end rate is $2/15$. In contrast, by solving OPT-noIN for this network instance, the optimal objective value is $1/15$. This means that the use of IN can increase the session throughput by 100% for this network instance.

C. Impact of Node Density

We present results of throughput gain of IN under difference node densities. For each randomly generated network instance, the throughput gain of IN is calculated by $(\hat{r}_{\min}^* - \tilde{r}_{\min}^*)/\tilde{r}_{\min}^*$, where \hat{r}_{\min}^* and \tilde{r}_{\min}^* are the optimal objective values from solving OPT-IN and OPT-noIN for this network instance, respectively. Figure 9 presents our simulation results. In the figure, the x -axis is the number of nodes in the network and the y -axis is the average throughput gain of IN over 100 randomly generated network instances. From Fig. 9, we can observe the significant throughput gain of IN: 57% for the 50-node networks; 74% for the 75-node networks; 81% for the 100-node networks; 85% for the 125-node networks; and 87% for the 150-node networks. Clearly, the throughput gain of IN increases with the node density. This can be explained by that the more nodes in the network, the more idle nodes are available to be used as neut. The larger pool of neut allows more interferences to be neutralized and thus allowing more transmissions to be carried out simultaneously. In the extreme case (not shown in the figures), when the node density becomes asymptotically large, all the interferences can be neutralized and the network operates in an “interference-free” regime.

VI. CONCLUSIONS

In this paper, we studied IN in a general wireless mesh network. We found that a multi-hop network environment (particularly a dense one) provides a fertile environment for us to unlock IN’s potential. By identifying eligible idle nodes as neut that can be exploited for IN, we established a mathematical framework for neut selection, IN, and scheduling. This mathematical framework allows us to determine a feasible space for a set of links that can be active simultaneously through optimal neut selection, IN among interference links, and scheduling. As an application, we applied this framework to study a throughput maximization problem and showed that IN can indeed boost throughput performance. In particular,

we found that the benefits of IN are most profound when the network is dense and there is a sufficient number of idle nodes that can be chosen for IN.

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