

# Opinions Power Opinions: Joint Link and Interaction Polarity Predictions in Signed Networks

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**Abstract**—Social media has been widely adopted by online users to share their opinions. Among users in signed networks, two types of opinions can be expressed. They can directly specify opinions to others via establishing positive or negative links; and they also can give opinions to content generated by others via a variety of social interactions such as commenting and rating. Intuitively these two types of opinions should be related. For example, users are likely to give positive (or negative) opinions to content from those with positive (or negative) links; and users tend to create positive (or negative) links with those that they frequently positively (or negatively) interact with. Therefore we can leverage one type of opinions to power the other. Meanwhile, they can enrich each other that can help mitigate the data sparsity and cold-start problems in the corresponding predictive tasks – link and interaction polarity predictions, respectively. In this paper, we investigate the problem of joint link and interaction polarity predictions in signed networks. We first understand the correlation between these two types of opinions; and then propose a framework that can predict signed links and the polarities of interactions simultaneously. The experimental results on a real-world signed network demonstrate the effectiveness of the proposed framework.

**Index Terms**—Signed Network, Social Media, Link Prediction, Interaction Polarity Prediction

## I. INTRODUCTION

Traditionally, network analysis has focused on unsigned networks (or networks with only positive links). Many social networks in social media can have positive and negative links (or signed networks [1], [2]). Such examples include the Epinions network with trust and distrust and the Slashdot network with friend and foe links. Negative links have been proven to advance various network analysis tasks such as link prediction [3]–[5], community detection [6], [7], and recommendations [8]. Therefore, signed networks are ubiquitous and have attracted increasing attention in recent years [9].

In signed networks, users can directly express positive (or negative) opinions to others by establishing positive (or negative) links. They can also specify positive (or negative) opinions to content created by others via various interactions such as commenting and rating. These two types of opinions should be related inherently. For example, a user receiving more positive (or negative) links is likely to receive more positive (or negative) opinions for his/her content; while users

IEEE/ACM ASONAM 2018, August 28–31, 2018, Barcelona, Spain  
 978-1-5386-6051-5/18/\$31.00 © 2018 IEEE

are likely to give positive (or negative) opinions to content generated by those with positive (or negative) links. Hence, link prediction [5] and interaction polarity prediction [10] are proposed to infer implicit opinions of these two types, respectively.

Recent years have witnessed a large number of algorithms for signed link prediction [4], [5], [11] and interaction polarity prediction [10], [12]–[14]. However, the majority of them have tackled these two tasks independently. As aforementioned, the corresponding opinions in these two tasks could be correlated and we could boost the performance by jointing these two tasks. Furthermore, since both tasks have been shown to severely suffer from the data sparsity problem [6], [14], by capturing the correlation between these two types of opinions, one can enrich the other. Hence, a joint framework has the potential to mitigate the data sparsity and cold-start problems for both tasks.

In this paper, we study the problem of joint link and interaction polarity predictions in signed networks. In particular, we create a framework that is able to harness these correlations for increased performance in both tasks. Our main contributions in this work have been summarized as follows:

- We propose a joint link and interaction prediction framework (LIP) that explicitly incorporates the correlations to predict links and interaction polarities simultaneously;
- We conduct experiments in a real-world signed network to demonstrate the effectiveness of LIP to the data sparsity problem.

The rest of this paper is organized as follows. In Section II we formally define the joint prediction problem. We discuss our proposed novel joint framework in Section III. In Section IV, we first introduce our dataset and then experimental results. We briefly review related work in Section V. Conclusions and future work are given in Section VI.

## II. PROBLEM

Let  $\mathcal{U} = \{u_1, u_2, \dots, u_n\}$  denote the set of  $n$  users. We represent signed links between users in an adjacency matrix,  $\mathbf{T} \in \mathbb{R}^{n \times n}$ , where  $\mathbf{T}_{ij} = 1$  if  $u_i$  creates a positive link to  $u_j$ ,  $-1$  if  $u_i$  creates a negative link to  $u_j$ , and  $0$  otherwise (i.e., when  $u_i$  has shown no link to  $u_j$ ). Let  $\mathcal{R} = \{r_1, r_2, \dots, r_m\}$  be the set of  $m$  content items generated by  $\mathcal{U}$ . We use

$\mathbf{A} \in \mathbb{R}^{n \times m}$  to denote the authorship matrix where  $\mathbf{A}_{ij} = 1$  if  $u_i$  creates  $r_j$  and  $\mathbf{A}_{ij} = 0$  otherwise. Social media provides multiple ways for users to express their opinions to content items generated by other users. For example, Facebook and Twitter allow their users to comment on content; Youtube provides thumbs-up and -down buttons; and Epinions enables its users to rate the helpfulness of the content with scores from 1 to 6. We use  $\mathbf{H} \in \mathbb{R}^{n \times m}$  to denote opinions expressed by  $\mathcal{U}$  to  $\mathcal{R}$ , where  $\mathbf{H}_{ik} = 1$  (or  $-1$ ) if  $u_i$  gives a positive (or negative) opinion to  $r_k$  and we use  $\mathbf{H}_{ik} = 0$  to indicate no explicit opinion is expressed from  $u_i$  to  $r_k$ . Note that in this paper, we define positive (or negative) interactions between  $u_i$  and  $u_j$  as  $u_i$  giving positive (or negative) opinions to content items generated by  $u_j$ . In other words, an interaction between users is defined as a triplet  $(u_i, r_k, u_j)$  where  $u_i$  expresses opinions to  $r_k$  that was generated by  $u_j$ .

With the above notations and definitions, our problem is stated as follows: *given the signed relations  $\mathbf{T}$ , the authorship matrix  $\mathbf{A}$  and the user-item opinion matrix  $\mathbf{H}$ , we aim to learn a predictor that can infer signed links and interaction polarities simultaneously by leveraging  $\mathbf{T}$ ,  $\mathbf{A}$ , and  $\mathbf{H}$ .*

### III. A FRAMEWORK FOR JOINT LINK AND INTERACTION POLARITY PREDICTIONS

We first validated our hypothesized existing correlations between a user's opinion of other users in regards to the links they form in signed social networks and the polarities of the interactions between them. Therefore, here we present our proposed framework LIP, which directly incorporates these found correlations into a joint optimization algorithm that can infer links and polarities of interactions jointly.

#### A. Basic Prediction Models

The low-rank matrix factorization approach has gained popularity recently across various applications such as link prediction [4] and recommender systems [14], [15]. In this work, we choose to build the basic prediction models based on the low-rank matrix factorization approach.

1) *Link Prediction:* Let  $\mathcal{T} = \{(u_i, u_j) | \mathbf{T}_{ij} \neq 0\}$  be the set of pairs with links. In terms of the link prediction task, we would like to find two latent matrices  $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_n] \in \mathbb{R}^{K_L \times n}$  and  $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n] \in \mathbb{R}^{K_L \times n}$ , with  $K_L$  being the number of latent dimensions, by solving the following optimization problem:

$$\min_{\mathbf{U}, \mathbf{V}} \frac{1}{2} \sum_{(u_i, u_j) \in \mathcal{T}} (\mathbf{T}_{ij} - \mathbf{u}_i^\top \mathbf{v}_j)^2 + \frac{\beta_1}{2} (\|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2) \quad (1)$$

where  $\mathbf{u}_i$  and  $\mathbf{v}_j$  are the user latent vectors representing giving and receiving link behaviors of  $u_i$ , respectively. Thus,  $\mathbf{u}_i^\top \mathbf{v}_j$  models the sign of a link from  $u_i$  to  $u_j$ , and therefore after optimizing the above formulation, we can use such inner products as a prediction for unknown user-user signed links in the network.

2) *Interaction Polarity Prediction:* Let  $\mathcal{H} = \{(u_i, r_k, u_j) | \mathbf{H}_{ik} \neq 0, \mathbf{A}_{jk} \neq 0\}$  be the set of interaction triplets and  $\mathbf{H}_{ik}$  denotes the opinion from  $u_i$  to the content  $r_k$  authored by  $u_j$ . Rather than taking the typical user-item

formulation, we instead want to formulate the model so that we can include information about the author of the content.

In this problem, we wish to find three latent matrices  $\mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_n] \in \mathbb{R}^{K_I \times n}$ ,  $\mathbf{Q} = [\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_n] \in \mathbb{R}^{K_I \times n}$ , and  $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_m] \in \mathbb{R}^{K_I \times m}$ , where  $\mathbf{p}_i$  and  $\mathbf{q}_i$  respectively denote the giving and receiving interaction behaviors of  $u_i$ , and  $\mathbf{s}_k$  is the latent vector for content  $r_k$ . These three matrices can be obtained via solving the following optimization problem:

$$\min_{\mathbf{P}, \mathbf{Q}, \mathbf{S}} \frac{1}{2} \sum_{(u_i, r_k, u_j) \in \mathcal{H}} (\mathbf{H}_{ik} - \mathbf{p}_i^\top (\mathbf{q}_j + \mathbf{s}_k))^2 + \frac{\beta_2}{2} (\|\mathbf{P}\|_F^2 + \|\mathbf{Q}\|_F^2 + \|\mathbf{S}\|_F^2) \quad (2)$$

#### B. Modeling Opinion Correlations

We first hypothesized and then validated that the giving (or receiving) behaviors in terms of links and interactions are correlated. Therefore, we can capture the opinion correlations by bridging the two giving behaviors via  $\mathbf{u}_i$  and  $\mathbf{p}_i$ , and the two receiving behaviors via  $\mathbf{v}_i$  and  $\mathbf{q}_i$ . Since the two giving behaviors are correlated, we can find a linear mapping matrix  $\mathbf{W}_O \in \mathbb{R}^{K_I \times K_L}$  that can map  $\mathbf{u}_i$  to  $\mathbf{p}_i$ . Given a set of latent vectors for all users  $u_i \in \mathcal{U}$ , it can then be easily seen that the linear mapping between them would be a solution to the following optimization problem:

$$\min_{\mathbf{W}_O} \sum_{u_i \in \mathcal{U}} \|\mathbf{W}_O \mathbf{u}_i - \mathbf{p}_i\|_2^2 \quad (3)$$

Similarly, we seek to find a matrix  $\mathbf{W}_I \in \mathbb{R}^{K_I \times K_L}$  to represent the mapping between  $\mathbf{v}_j$ , and  $\mathbf{q}_j$ . The mapping  $\mathbf{W}_I$  can be learned as follows:

$$\min_{\mathbf{W}_I} \sum_{u_j \in \mathcal{U}} \|\mathbf{W}_I \mathbf{v}_j - \mathbf{q}_j\|_2^2 \quad (4)$$

Eqs. (3) and (4) can capture opinion correlations for links and interactions. They also allow us to bridge the two basic models for link and interaction polarity predictions together.

#### C. The Proposed Joint Framework

Given the above formulations, we now present our joint framework LIP that solves the following optimization problem:

$$\begin{aligned} & \min_{\substack{\mathbf{U}, \mathbf{V}, \mathbf{P}, \mathbf{Q}, \\ \mathbf{S}, \mathbf{W}_I, \mathbf{W}_O}} \mathcal{L}(\mathbf{U}, \mathbf{V}, \mathbf{P}, \mathbf{Q}, \mathbf{S}, \mathbf{W}_I, \mathbf{W}_O) \\ &= \frac{1}{2} \sum_{(u_i, u_j) \in \mathcal{T}} (\mathbf{T}_{ij} - \mathbf{u}_i^\top \mathbf{v}_j)^2 \\ &+ \frac{\eta}{2} \sum_{(u_i, r_k, u_j) \in \mathcal{H}} (\mathbf{H}_{ik} - \mathbf{p}_i^\top (\mathbf{q}_j + \mathbf{s}_k))^2 \\ &+ \frac{\gamma}{2} \left( \sum_{u_i \in \mathcal{U}} \|\mathbf{W}_O \mathbf{u}_i - \mathbf{p}_i\|_2^2 + \sum_{u_j \in \mathcal{U}} \|\mathbf{W}_I \mathbf{v}_j - \mathbf{q}_j\|_2^2 \right) \\ &+ \frac{\beta_1}{2} (\|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2) + \frac{\beta_2}{2} (\|\mathbf{P}\|_F^2 + \|\mathbf{Q}\|_F^2 + \|\mathbf{S}\|_F^2) \\ &+ \frac{\beta_3}{2} (\|\mathbf{W}_I\|_F^2 + \|\mathbf{W}_O\|_F^2) \end{aligned} \quad (5)$$

where the first term is a standard user-user matrix factorization model for the link prediction problem. The second term is a modification to the user-review matrix factorization model that also incorporates the additional vector  $\mathbf{q}_j \forall u_j \in \mathcal{U}$  to represent the influence of the author  $u_j$  in the prediction of  $u_i$ 's opinion on  $r_k$ , when  $r_k$  was written by  $u_j$ . The third and fourth terms capture the correlations of giving and receiving behaviors, respectively, and their contributions are controlled by a parameter  $\gamma$ . Other terms in Eq. (5) are added to avoid overfitting.

We note that the balance between optimizing for the two tasks (signed link prediction and user interactions polarities) is balanced by the parameter  $\eta$ , where a small increase in this value will result in an increase to the importance of the user interaction polarity prediction task. If a user  $u_i$  has no link information, they are deemed a cold-start user in the link prediction task. Thus there is no way to learn  $\mathbf{u}_i$  and  $\mathbf{v}_i$  in the basic model and we fail to do link prediction for  $u_i$ . However, if  $u_i$  has had some interactions with other users in the network, we can learn  $\mathbf{p}_i$  and  $\mathbf{q}_i$  from his/her interaction data. Thus, the proposed framework LIP can also learn  $\mathbf{u}_i$  and  $\mathbf{v}_i$  via the model components of capturing giving and receiving correlations. Similarly, LIP can also help when  $u_i$  has no interaction data but has link information. Via the above analysis, we note that LIP has the potential to mitigate the data sparsity and cold-start problems in either link prediction or interaction polarity prediction.

We use stochastic gradient descent (SGD) to solve the optimization problem shown above. We use the combined training data  $\mathcal{X} = \{\mathcal{T} \cup \mathcal{H}\}$ , where  $\mathcal{T}$  and  $\mathcal{H}$  are the link and interaction training data, respectively.

#### IV. EXPERIMENTS

In this section, we first introduce our dataset and then conduct experiments to answer the question: Can our joint model help alleviate the sparsity problem in these two prediction tasks? To address this question, we perform experiments in which we increase the sparsity of the training data and compare the performance with representative baselines.

##### A. Dataset

We collected a dataset from Epinions for this investigation. Epinions users can give positive and negative links to each other (i.e.,  $\mathbf{T}$  matrix). They also can write reviews and we use this data to construct the authorship matrix  $\mathbf{A}$ . Reviews can use scores from 1 to 6 to indicate the helpfulness by others that we use these to construct the matrix  $\mathbf{H}$  (and we use  $\{4, 5, 6\}$  and  $\{1, 2, 3\}$ , to be the positive and negative ratings respectively. Some statistics of the dataset are shown in Table I. The task of creating (or receiving) a signed link to others can be thought of as an explicit form of expressing one’s opinion of (or from) others. In contrast, when a user interacts with the content authored by others, they are implicitly marking their opinion towards others in these interactions. Therefore, it is reasonable to assume that the implicit and explicit opinions among users are correlated. Next we investigate these correlations from both global and local perspectives.

##### B. Experimental Settings

We perform a filtering of the data that only keeps users that have both given and received a link, and also requires the users to have given at least one helpfulness rating and have also authored at least one review that has received at least one helpfulness rating. After the filtering process, we were left with 29,901 users, 600,976 user-user links, and 11,555,599 helpfulness ratings. The dataset has been randomly split into

TABLE I  
EPINIONS DATASET STATISTICS.

# of Users	233,429
# of Positive Links	717,667
# of Negative Links	123,705
Density of $\mathbf{T}$	$7.75 \times 10^{-5}$
# of Reviews	755,722
# of Positive Interactions	12,581,553
# of Negative Interactions	1,086,551
Density of $\mathbf{H}$	$1.54 \times 10^{-5}$

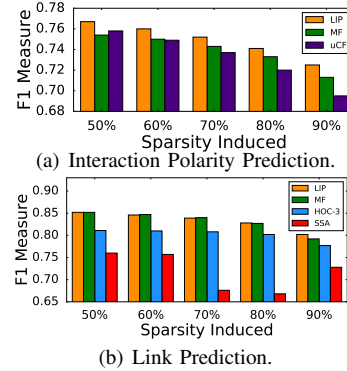


Fig. 1. Results with varied Sparsity Settings.

70% for training, 10% for validation, and 20% for testing. Note that we then balanced our testing dataset to be 50% positive and 50% negative similar to that done in [5]. To evaluate and compare the performance of LIP, we present the F1 measure for the interaction polarity and the link prediction tasks. Note that the higher the value, the better the performance.

For all the models that required parameters to be tuned, we used the validation set to obtain the best parameters for each respective model.

##### C. Performance Comparison on Sparsity Experiments

To answer the our posed question, we compare the proposed framework, LIP, with existing interaction polarity and link prediction methods. We choose the following representative interaction polarity prediction baselines for comparison:

- *uCF*: User-based collaborative filtering (k=5, cosine similarity) based on their helpfulness ratings. For details on collaborative filtering please see [16].
- *MF*: Our matrix factorization method (i.e., Eq. (2)).

For signed link prediction, the representative baselines are the following:

- *SSA*: A signed spectral method [7] using regularized Laplacian kernel [17] after making  $\mathbf{T}$  symmetric.
- *HOC-3*: A supervised approach [5] that uses 16 directed triad configurations and 7 node related features.
- *MF*: Matrix factorization method [4] shown in Eq. (1).

In this experiment, we simulate a ranging sparsity across each user by removing x% of the links and interactions for each user and vary x in  $\{50, 60, 70, 80, 90\}$ .

##### D. Experimental Results

The interaction polarity prediction results can be found in Figure 1(a). Most of the time we have LIP finding significant gains over MF across the levels of sparsity induced.

The sparsity experiments results for the link prediction are in Figure 1(b). LIP and MF obtain much better performance than SSA and HOC-3. We are able to observe that LIP performs comparable to the MF method for the lower sparsity settings, but upon reaching the higher sparsity level, LIP achieves better performance than MF.

From the results in the sparsity experiment, we have seen LIP’s ability to help alleviate the sparsity problem. More specifically, we see a significant improvement in the interaction polarity predictions, and increasing improvement for the link prediction with the increase of the sparsity. We can therefore reason that the linking terms based upon the correlated user opinions in LIP are able to provide a transfer of information between the two tasks that ultimately have a user’s opinions in one task power the other. Based upon the results presented in this section we have shown that indeed LIP is able to utilize the inherent correlations behind the opinions expressed in the two tasks to boost the performance in both the prediction tasks simultaneously.

## V. RELATED WORK

Although there has been a large number of recent works focused on signed link prediction and even interaction polarity prediction, most of their major drawbacks have been that they optimized each task one at a time. Previous work on link prediction in signed networks can be split into two primary categories; supervised [5], [11] and unsupervised methods [4].

The literature on the interaction polarity prediction is quite limited in comparison to the classical link prediction task. It was in [10] that the authors had the objective of specifically attempting to predict the rating a user would give the content generated by another user. Unlike our work, they included information about the content of the reviews. In [14] they used the interactions for increased performance in recommendations to the users. In [13] personalized predictions for review helpfulness were made using a tensor factorization model.

## VI. CONCLUSION AND FUTURE WORK

In signed networks, users can express their opinions via two activities, i.e., creating signed links and expressing opinions on the content from others. Intuitively, the opinions and behaviors that the users have when performing these two activities online should be related. Our proposed joint optimization framework, LIP, for the prediction of signed links and interaction polarities that was built upon having the opinions in one task power the other. This novel framework was able to boost the performance in both prediction tasks when jointly solving the two problems as compared to separately solving them individually. The significance becomes even more important in settings where the social network data is sparse. This is due to the fact that LIP is able to partially avoid and mitigate the sparsity problem by transferring information about users opinions from one problem to another by capturing the correlations between them. Our experiments on a real-world signed network have demonstrated the effectiveness of LIP and also its robustness to the data sparsity problem.

Future work in this domain will be to seek other problems that users might have correlated opinions or behaviors that can be harnessed to increase the performance in multiple tasks simultaneously. We also would like to investigate the underlying dynamics in signed networks that are causing these correlations, or other phenomenon, such as high reciprocity in some networks and not in others.

## ACKNOWLEDGMENT

This work is supported by the National Science Foundation (NSF) under grant number IIS-1714741 and IIS-1715940.

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