Homework 1 – 50 points

Quantitative Comparison of Packet Switching and Circuit Switching – 20 points

Consider the two scenarios below:

- A circuit-switching scenario in which \( N_{cs} \) users, each requiring a bandwidth of 25 Mbps, must share a link of capacity 100 Mbps.
- A packet-switching scenario with \( N_{ps} \) users sharing a 100 Mbps link, where each user again requires 25 Mbps when transmitting, but only needs to transmit 10 percent of the time.

Answer the following questions:

1. When circuit switching is used, what is the maximum number of circuit-switched users that can be supported? Explain your answer.
2. For the remainder of this problem, suppose packet switching is used. Suppose there are 7 packet-switching users (i.e., \( N_{ps} = 7 \)). Can this many users be supported under circuit-switching? Explain.
3. What is the probability that a given (specific) user is transmitting, and the remaining users are not transmitting?
4. What is the probability that one user (any one among the 7 users) is transmitting, and the remaining users are not transmitting? When one user is transmitting, what fraction of the link capacity will be used by this user?
5. What is the probability that any 4 users (of the total 7 users) are transmitting and the remaining users are not transmitting? (Hint: you will need to use the binomial distribution \(^\text{12}\)).
6. What is the probability that more than 4 users are transmitting? Comment on what this implies about the number of users supportable under circuit switching and packet switching.

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1. [https://www.youtube.com/watch?v=O12yTz_8EOw](https://www.youtube.com/watch?v=O12yTz_8EOw)
Solution:

1. When circuit switching is used, at most 4 circuit-switched users that can be supported. This is because each circuit-switched user must be allocated its 25 Mbps bandwidth, and there is 100 Mbps of link capacity that can be allocated.

2. No. Under circuit switching, the 7 users would each need to be allocated 25 Mbps, for an aggregate of 175 Mbps - more than the 100 Mbps of link capacity available.

3. The probability that a given (specific) user is busy transmitting, which we'll denote $p$, is just the fraction of time it is transmitting, i.e., 0.100. The probability that one specific other user is not busy is $(1-p)$, and so the probability that all of the other $N_{ps}-1$ users are not transmitting is $(1-p)^{N_{ps}-1}$. Thus the probability that one specific user is transmitting and the remaining users are not transmitting is $p(1-p)^{N_{ps}-1}$, which has the numerical value of 0.0531441. This user will be transmitting at a rate of 25 Mbps over the 100 Mbps link, using a fraction 0.2500 of the link's capacity when busy.

4. The probability that exactly one (any one) of the $N_{ps}$ users is busy is $N_{ps}$ times the probability that a given specific user is transmitting and the remaining users are not transmitting (our answer to (3) above), since the one transmitting user can be any one of the $N_{ps}$ users. The answer to (4) is thus $N_{ps}p^1(1-p)^{N_{ps}-1}$, which has the numerical value of 0.3720087.

5. The probability that 4 specific users of the total 7 users are transmitting and the other 3 users are idle is $p^4(1-p)^3$. Thus the probability that any 4 of the 7 users are busy is \( \text{choose}(7,4)p^4(1-p)^3 \), where \( \text{choose}(7,4) \) is the \( (7,4) \) coefficient of the binomial distribution, i.e. the number combinations of picking 4 items of 7. The numerical value of this probability is 0.0025515.

6. The probability that more than 4 users of the total 7 users are transmitting is \( \sum_{i=5}^{7} \text{choose}(7,i)p^i(1-p)^{7-i} \). The numerical value of this probability is 0.0001765. Note that 4 is the maximum number of users that can be supported using circuit switching (the answer to part (1)). With packet switching, nearly twice as many users (7) are supported with a small probability that more than 4 of these packet-switching users are busy at the same time.
Computing the one-hop transmission delay – 10 points

Consider the figure below, in which a single router is transmitting packets, each of length L bits, over a single link with transmission rate R Mbps to another router at the other end of the link.

Suppose that the packet length is L = 4000 bits, and that the link transmission rate along the link to router on the right is R = 1000 Mbps.

1. What is the transmission delay (the time needed to transmit all of a packet's bits into the link)?
2. What is the maximum number of packets per second that can be transmitted by the link?

Solution:

1. The link transmission delay = L/R = 4000 bits / 1000 Mbps = 0.004000 msec.
2. The link can transmit 250000.000000 packets per second

Computing end-end delay (transmission and propagation delay) – 10 points

Consider the figure below, with three links, each with the specified transmission rate and link length.

Find the end-to-end delay (including the transmission delays and propagation delays on each of the three links, but ignoring queueing delays and processing delays) from when the left host begins transmitting the first bit of a packet to the time when the last bit of that packet is received at the server at the right. The speed of light propagation delay on each link is 3x10**8 m/sec. Note that the transmission rates are in Mbps and the link distances are in Km. Assume a packet length of 12000 bits. Give your answer in milliseconds.
Solution:

- Link 1 transmission delay = \( \frac{L}{R} = \frac{12000 \text{ bits}}{1000 \text{ Mbps}} = 0.012000 \text{ msec.} \)
- Link 1 propagation delay = \( \frac{d}{s} = \frac{2 \text{ Km}}{3 \times 10^8 \text{ m/sec}} = 0.006667 \text{ msec.} \)
- Link 2 transmission delay = \( \frac{L}{R} = \frac{12000 \text{ bits}}{1000 \text{ Mbps}} = 0.012000 \text{ msec.} \)
- Link 2 propagation delay = \( \frac{d}{s} = \frac{500 \text{ Km}}{3 \times 10^8 \text{ m/sec}} = 1.666667 \text{ msec.} \)
- Link 3 transmission delay = \( \frac{L}{R} = \frac{12000 \text{ bits}}{1000 \text{ Mbps}} = 0.012000 \text{ msec.} \)
- Link 3 propagation delay = \( \frac{d}{s} = \frac{1 \text{ Km}}{3 \times 10^8 \text{ m/sec}} = 0.003333 \text{ msec.} \)

Thus, the total end-to-end delay is the sum of these six delays: 1.712667 msecs.
End to End Throughput and Bottleneck Links – 10 points

Consider the scenario shown below, with four different servers connected to four different clients over four three-hop paths. The four pairs share a common middle hop with a transmission capacity of $R = 300$ Mbps. The four links from the servers to the shared link have a transmission capacity of $R_S = 40$ Mbps. Each of the four links from the shared middle link to a client has a transmission capacity of $R_C = 90$ Mbps per second. You might want to review Figure 1.20 in the text before answering the following questions:

1. What is the maximum achievable end-end throughput (in Mbps) for each of four client-to-server pairs, assuming that the middle link is fair-shared (i.e., divides its transmission rate equally among the four pairs)?
2. Which link is the bottleneck link for each session?
3. Assuming that the senders are sending at the maximum rate possible, what are the link utilizations for the sender links ($R_S$), client links ($R_C$), and the middle link ($R$)?

Solution:

1. The maximum achievable end-end-throughput is 40 Mbps.
2. This is the transmission capacity of the first hop, which is the bottleneck link, since the first-hop transmission capacity of 40 Mbps is less than one quarter of the shared-link transmission capacity ($300/4 = 75$ Mbps) and less than the third-hop transmission capacity of 90 Mbps.
3. The utilization of sender links is 100% (40/40 = 100%). The utilization of client links is 44.44% (40/90 = 44.44%). The utilization of the middle link is 53.33% (40/75 or $4*40/300 = 53.33\%$).