

Accelerated Runtime Verification of LTL Specifications with Counting Semantics

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Abstract. This paper presents a novel and efficient parallel algorithm for runtime verification of an extension of LTL that allows for nested quantifiers subject to numerical constraints. Such constraints are useful in evaluating thresholds (e.g., expected uptime of a web server). Our algorithm uses the *MapReduce* programming model to split a program trace into variable-based clusters at run time. Each cluster is then mapped to its respective monitor instances, verified, and reduced collectively on a multi-core CPU or the GPU. Our experiments on real-world case studies show that the algorithm imposes negligible monitoring overhead.

1 Introduction

Runtime verification (RV) is an automated specification-based technique, where a *monitor* evaluates the correctness of a set of logical properties on a particular execution. RV complements exhaustive approaches such as model checking and theorem proving and under-approximated methods such as testing. RV can be particularly helpful in scenarios, where one needs to monitor parametric requirements on types of execution entities (e.g., processes and threads), user- and kernel-level events and objects (e.g., locks, files, sockets), web services (e.g., requests and responses), and network traffic. For example, the requirement ‘every open file should eventually be closed’ specifies a rule for causal and temporal order of opening and closing individual objects which generalizes to *all* files. Such properties can become even more complex by incorporating numerical constraints such as thresholds, floors, ceilings. However, to our knowledge existing RV frameworks fall short in expressing counting semantics.

In this paper, we extend the 4-valued semantics of LTL (denoted RV-LTL in this paper) [6] by adding counting semantics with numerical constraints and propose an efficient parallel algorithm for their verification at run time. Inspired by the work in [15], the syntax of our language (denoted LTL₄-C) extends LTL syntax by the addition of *counting quantifiers*. That is, we introduce two quantifiers: the *instance counting quantifier* (\mathbb{E}) which allows expressing properties that reason about the number of satisfied or violated instances, and the *percentage counting quantifier* (\mathbb{A}) which allows reasoning about the percentage of satisfied

or violated instances out of all instances in a trace. These quantifiers are subscripted with numerical constraints to express the conditions used to evaluate the count. For example, the following LTL_4-C formula:

$$\mathbb{A}_{\geq 0.95} s : \text{socket}(s) \cdot (\mathbf{G} \text{ receive}(s) \Rightarrow \mathbf{F} \text{ respond}(s))$$

intends to express the property that ‘at least 95% of TCP/UDP sockets must eventually respond to a received request’. For a web admin, ideally the number of dropped requests is zero, however in reality requests will be dropped sometimes [20]. Thus it is beneficial for a monitor to keep track of the percentage of dropped requests and fire an alert once a certain threshold is exceeded.

Also, the formula:

$$\mathbb{A}x : \text{user}(x) \cdot (\mathbb{E}_{\leq 3} r : \text{rid}(r) \cdot (\text{login}(x, r) \wedge \text{unauthorized}(x, r)))$$

intends to capture the requirement that ‘for all users, there exist at most 3 requests of type login that end with an unauthorized status’. Similar to RV-LTL, the semantics of LTL_4-C is defined over four truth values: *permanently true/false* in addition to *presumably true/false*. Presumably true/false is designed to support monitoring of partial traces, where a future extension of the trace could potentially violate/satisfy the property. An example of such a property is the socket response property above. If a request is responded to at a point in time, the property is presumably satisfied, yet could be violated if in the future the socket receives a request and never responds to it afterwards. Without presumably true/false, one could not extract an informative verdict about such a basic property, due to the unavoidable way it is semantically built.

The first contribution of the paper is extending RV-LTL by redefining presumably true/false within the context of counting semantics. Consider the example demonstrated above, where it is required that a web server drops less than 5% of the requests. For such a property, counting semantics justify the need for presumably true/false in a similar fashion to incomplete executions in RV-LTL. For instance, if only 4% of the requests have been dropped so far, that does not mean that the property is permanently satisfied. There could be more requests that arrive in the future and are dropped, increasing the percentage of dropped requests beyond 5%. A verdict of true is incorrect, since the property can be violated in the future. In LTL_4-C , this property is presumably satisfied, with a potential to be violated by more requests. For a web admin, this verdict indicates that the system is currently healthy.

The second contribution of this paper is a divide-and-conquer-based online monitor generation technique for LTL_4-C specifications. Our technique first synthesizes an RV-LTL monitor for the inner LTL formula of the given LTL_4-C formula at pre-compile time using the technique in [6]. Then, based upon the values of variables observed at run time, submonitors are generated and merged to compute the current truth value of a property for the current program trace.

Our third contribution is a monitoring algorithm that implements the above approach for verification of LTL_4-C properties at run time. The monitoring algorithm evaluates properties in parallel, utilizing multicore CPUs or GPUs and

maximizing the throughput of the monitor. The algorithm utilizes the popular *MapReduce* programming model to (1) spawn submonitors that aim at evaluating subformulas using partial quantifier elimination, and (2) merge partial evaluations to compute the current truth value of properties.

Our parallel algorithm for verification of LTL_4-C properties is fully implemented on multi-core CPU and GPU technologies using our own simple implementation of the MapReduce programming model. We report experimental results by conducting three real-world independent case studies. The first case study is a monitor for HTTP requests and responses on an Apache Web Server. The second case study is a monitor for upload chunk size based on a dataset for profiling DropBox traffic. The third case study monitors a network proxy cache to reduce the bandwidth usage of online video services, based on a YouTube request dataset. We present performance results comparing single-core CPU, multi-core CPU, and GPU implementations. Our results show that our GPU-based implementation provides an average speed up of 6.3x when compared to single-core CPU, and 1.75x when compared to multi-core CPU. The CPU utilization of the GPU-based implementation is negligible compared to multi-core CPU, freeing up the system to perform more computation. Thus, the GPU-based implementation manages to provide competitive speedup while maintaining a low CPU utilization, which are two goals that the CPU cannot achieve at the same time. Put it another way, the GPU-based implementation incurs minimal monitoring costs while maintaining a high throughput.

Organization Section 2 describes the syntax and semantics of LTL_4-C . In Section 3, we explain our online monitoring approach, while Section 4 presents our parallelization technique based on MapReduce. Experimental results are presented in Section 5. Related work is discussed in Section 6. Finally, we make concluding remarks and discuss future work in Section 7.

2 LTL with counting semantics

Let IP be a finite set of interpreted predicates, and let $\Sigma = 2^{IP}$ be the power set of IP . We call each element of Σ an *event*.

Definition 1 (Trace). *A trace $w = w_0w_1 \dots$ is a finite or infinite sequence of events where each event consists of interpreted predicates; i.e., $w_i \in \Sigma$, for all $i \geq 0$. ■*

We denote the set of all infinite traces by Σ^ω and the set of all finite traces by Σ^* .

2.1 Syntax of LTL_4-C

LTL_4-C extends RV-LTL [6] (also known as RV-LTL) with two counting quantifiers: the instance counting quantifier (\mathbb{E}) and the percentage counting quantifier (\mathbb{A}). The semantics of these quantifiers are introduced in subsection 2.4. The syntax of LTL_4-C is defined as follows:

Definition 2 (LTL₄-C Syntax). LTL₄-C formulas are defined using the following grammar:

$$\begin{aligned} \varphi &::= \mathbb{A}_{\sim k} x : p(x) \cdot \varphi \mid \mathbb{E}_{\sim l} x : p(x) \cdot \varphi \mid \psi \\ \psi &::= \top \mid a \mid p(x_1 \cdots x_n) \mid \neg\psi \mid \psi_1 \wedge \psi_2 \mid \mathbf{X}\psi \mid \psi_1 \mathbf{U} \psi_2 \end{aligned}$$

where \mathbb{A} is the percentage counting quantifier, \mathbb{E} is the instance counting quantifier, $x, x_1 \cdots x_n$ are variables with finite domains $\mathcal{D}, \mathcal{D}_1, \cdots, \mathcal{D}_n$, $\sim \in \{<, \leq, >, \geq, =\}$, $k: \mathbb{R} \in [0, 1]$, $l \in \mathbb{Z}^+$, a is an atomic proposition, \mathbf{X} is the next, and \mathbf{U} is the until temporal operators. ■

If we omit the numerical constraint in $\mathbb{A}_{\sim k}$ (respectively, $\mathbb{E}_{\sim l}$), we mean $\mathbb{A}_{=1}$ (respectively, $\mathbb{E}_{\geq 1}$). The syntax of LTL₄-C forces constructing formulas, where a string of counting quantifiers is followed by a quantifier-free formula. We emphasize that \mathbb{A} and \mathbb{E} do not necessarily resemble standard first order quantifiers \forall and \exists . In fact, $\neg\mathbb{A}$ and \mathbb{E} are not equivalent.

Consider the LTL₄-C property $\varphi = \mathbb{A}x : p(x) \cdot \psi$, where the domain of x is \mathcal{D} . This property denotes that for any possible valuation of the variable x ($[x := v]$), if $p(v)$ holds, then ψ should hold. If $p(v)$ does not hold, then $p(v) \cdot \psi$ evaluates to true. This effectively means that the quantifier $\mathbb{A}x$ is in fact applied only over the sub-domain $\{v \in \mathcal{D} \mid p(v)\} \subseteq \mathcal{D}$.

To give an intuition, consider the scenarios where file management anomalies can cause serious problems at run time (e.g., in NASA’s Spirit Rover on Mars in 2004). For example, the following LTL₄-C property expresses “at least half of the files that a process has previously opened must be closed”:

$$\varphi = \mathbb{A}_{\geq 50\%} f : \text{inevent}(f) \cdot \mathbf{G}(\text{opened}(f) \mathbf{U} \text{close}(f)) \quad (1)$$

where inevent is the p predicate of the quantifier, denoting that the concrete file appeared in an event in the trace.

2.2 Truth Values of LTL₄-C

The objective of LTL₄-C is to verify the correctness of quantified properties at run time with respect to finite program traces. Such verification attempts to produce a sound verdict regardless of future continuations.

Similar to RV-LTL, we incorporate four truth values to define the semantics of LTL₄-C: $\mathbb{B}_4 = \{\top, \perp, \top_p, \perp_p\}$; *true*, *false*, *presumably true*, and *presumably false*, respectively. The values in \mathbb{B}_4 form a lattice ordered as follows: $\perp < \perp_p < \top_p < \top$. Given a finite trace u and an LTL₄-C property φ , the informal description of evaluation of u with respect to φ is as follows:

- **True** (\top) denotes that any infinite extension of u satisfies φ . For example, $\varphi_1 = \mathbb{E}_{\geq 1} t : \text{thread}(t) \cdot \log(t)$ is a property that checks a process has at least one log thread. If one log thread is found in the trace, the property is permanently satisfied.

- **False** (\perp) denotes that any infinite extension of u violates φ . For example, $\varphi_2 = \mathbb{E}_{=1}t : \text{thread}(t) \cdot \log(t)$ is a property that checks a process has *exactly* one log thread. If more than one log thread is found in the trace, the property is permanently violated.
- **Presumably true** (\top_p) extends the definition of *presumably true* in RV-LTL [6], where \top_p denotes that u satisfies the inner LTL property and the counting quantifier constraint in φ , if the program terminates after execution of u . An example is

$$\varphi_3 = \mathbb{E}_{\geq 1}t : \text{thread}(t) \cdot \log(t) \wedge \mathbf{G}(\text{event}(t) \Rightarrow \mathbf{F}\text{write}(t))$$

which evaluates to \top_p if there is only one log thread that has received an event and has written it, but can still potentially receive another event and never write it, thus violating the property.

- **Presumably false** (\perp_p) extends the definition of *presumably false* in RV-LTL [6], which denotes that u presumably violates the quantifier constraint in φ . For example, Property φ_3 evaluates to \perp_p if there is one log thread that has received an event and not yet written it. A future extension of the trace can potentially contain a write event, thus transforming the valuation of the property to \top_p .

2.3 Valuation in $\text{LTL}_4\text{-C}$

An $\text{LTL}_4\text{-C}$ property essentially defines a set of traces, where each trace is a sequence of events (i.e., sets of predicates). We define the semantics of $\text{LTL}_4\text{-C}$ with respect to finite traces and present a method of utilizing these semantics for runtime verification. In the context of runtime verification, the objective is to ensure that a trace is in the set of traces that the property defines.

To introduce the semantics of $\text{LTL}_4\text{-C}$, we examine counting quantifiers further. Since the syntax of $\text{LTL}_4\text{-C}$ allows nesting of counting quantifiers, a canonical form of properties is $\varphi = \mathbb{Q}_\varphi \psi$ where ψ is an LTL property and \mathbb{Q}_φ is a sequence of counting quantifiers $\mathbb{Q}_\varphi = \mathbb{Q}_0\mathbb{Q}_1 \cdots \mathbb{Q}_{n-1}$ such that each $\mathbb{Q}_i = \langle q_i, \sim_i, c_i, x_i, p_i \rangle$, $0 \leq i \leq n-1$, is a tuple encapsulating the counting quantifier information. That is, $q_i \in \{\mathbb{A}, \mathbb{E}\}$, $\sim_i \in \{<, \leq, >, \geq, =\}$, c_i is the constraint constant, x_i is the bound variable, and p_i is the predicate within the quantifier (see Definition 2).

Variable Valuation We define a vector D_φ with respect to a property φ as $D_\varphi = \langle d_0, d_1, \dots, d_{n-1} \rangle$ where $n = |\mathbb{Q}_\varphi|$ and d_i , $0 \leq i \leq n-1$, is a value for variable x_i . We denote the first m components of the vector D_φ (i.e., $\langle d_0, d_1, \dots, d_{m-1} \rangle$) by $D_\varphi|^m$. We refer to D_φ as a *value vector* and to $D_\varphi|^m$ as a *partial value vector*.

A *property instance* $\hat{\varphi}(D_\varphi|^m)$ is obtained by replacing every occurrence of the variables $x_0 \cdots x_{m-1}$ in φ with the values $d_0 \cdots d_{m-1}$, respectively. Thus, $\hat{\varphi}(D_\varphi|^m)$ is free of quantifiers of index less than m , yet remains quantified over

variables $x_m \cdots x_{n-1}$. $\hat{\varphi}(D_\varphi)$ denotes replacing all quantified variables with values in D_φ , resulting in an unquantified LTL property. For instance, for the following property $\varphi = \mathbb{A}_{>c_1} x : p_x(x) \cdot (\mathbb{A}_{<c_2} y : p_y(y) \cdot \mathbf{G} q(x, y))$ and value vector $D_\varphi = \langle 1, 2 \rangle$ (i.e., the vector of values for variables x and y , respectively), $\hat{\varphi}(D_\varphi)$ will be $\hat{\varphi}(\langle 1, 2 \rangle) = p_x(1) \cdot (p_y(2) \cdot \mathbf{G} q(1, 2)) = \mathbf{G} q(1, 2)$.

We now define the set $\mathbb{D}_{\varphi, u}$ as the set of all value vectors with respect to a property $\varphi = \mathbb{Q}_\varphi \psi$ and a trace $u = u_0 u_1 \cdots$:

$$\mathbb{D}_{\varphi, u} = \{D_\varphi \mid \exists j \geq 0 : \forall i \in [0, |\mathbb{Q}_\varphi|) : p_i(d_i) \in u_j\} \quad (2)$$

Valuation of Property Instances As per the definition of $\mathbb{D}_{\varphi, u}$, every value vector $D_\varphi = \langle d_0 \cdots d_{n-1} \rangle$ in $\mathbb{D}_{\varphi, u}$ contains values for which the predicates $p_i(d_i)$ hold in some trace event u_j . For simplicity, we denote this as a value vector *in* a trace event u_j . These value vectors can possibly be in multiple and interleaved events in the trace. Thus, we define a trace $u^{D_\varphi} = u_0^{D_\varphi} u_1^{D_\varphi} \cdots$ as a subsequence of the trace u such that the value vector D_φ is in every event:

$$\forall j \geq 0 : \forall i \in [0, n-1] : p_i(d_i) \in u_j^{D_\varphi}$$

2.4 Semantics of LTL4-C

Definition 3 (LTL₄-C Satisfaction Relation). *Given an LTL₄-C property $\varphi = \mathbb{Q} \psi$ where \mathbb{Q} is a quantifier (either \mathbb{A} or \mathbb{E}), and ψ is an LTL₄-C formula. Also, given an infinite trace w , we define the satisfaction relation $w \models_4 \varphi$ as follows:*

$$\begin{aligned} w \models_4 \psi & \quad \text{iff } w \models \psi \text{ and } \psi \text{ is an LTL property} \\ w \models_4 \mathbb{E}_{\sim c} x : p_x(x) \cdot \psi & \quad \text{iff } \exists \mathbb{D}'_{\varphi, w} \subset \mathbb{D}_{\varphi, w} \text{ s.t. } \forall D_\varphi \in \mathbb{D}'_{\varphi, w} : w^{D_\varphi} \models \hat{\varphi}(D_\varphi) \wedge \\ & \quad \forall D_\varphi \notin \mathbb{D}'_{\varphi, w} : w^{D_\varphi} \not\models \hat{\varphi}(D_\varphi) \wedge |\mathbb{D}'_{\varphi, w}| \sim c \\ w \models_4 \mathbb{A}_{\sim c} x : p_x(x) \cdot \psi & \quad \text{iff } \exists \mathbb{D}'_{\varphi, w} \subset \mathbb{D}_{\varphi, w} \text{ s.t. } \forall D_\varphi \in \mathbb{D}'_{\varphi, w} : w^{D_\varphi} \models \hat{\varphi}(D_\varphi) \wedge \\ & \quad \forall D_\varphi \notin \mathbb{D}'_{\varphi, w} : w^{D_\varphi} \not\models \hat{\varphi}(D_\varphi) \wedge |\mathbb{D}'_{\varphi, w}| / |\mathbb{D}_{\varphi, w}| \sim c \end{aligned}$$

where $\mathbb{D}_{\varphi, w}$ is the finite set of all value vectors in the infinite trace w . $\hat{\varphi}(D_\varphi)$ is an LTL property and \models is the satisfaction relation as defined in LTL semantics.

Definition 4 (LTL₄-C Semantics for finite prefixes). *Given an LTL₄-C property $\varphi = \mathbb{Q}_{\sim c} \psi$ where \mathbb{Q} is a quantifier and ψ is an LTL₄-C formula. Also,*

given a finite prefix u of a trace, $\text{LTL}_4\text{-C}$ semantics are defined as follows:

$$[u \models_4 \varphi] = \begin{cases} [u \models_{\text{RV-LTL}} \varphi] & \text{iff } \varphi \text{ is an LTL property} \\ \top & \text{iff } \exists \mathbb{D}'_{\varphi,u} \subset \mathbb{D}_{\varphi,u} \text{ s.t.} \\ & \forall D_\varphi \in \mathbb{D}'_{\varphi,u} : [u^{D_\varphi} \models_4 \hat{\varphi}(D_\varphi)] = \top \wedge \\ & \forall D_\varphi \notin \mathbb{D}'_{\varphi,u} : [u^{D_\varphi} \models_4 \hat{\varphi}(D_\varphi)] \neq \top \wedge \\ & |\mathbb{D}'_{\varphi,u}| \sim c \text{ if } \mathcal{Q} = \mathbb{E} \text{ else } |\mathbb{D}'_{\varphi,u}|/|\mathbb{D}_{\varphi,u}| \sim c \wedge \\ & \forall v \in \Sigma^\omega : uv \models_4 \varphi \\ \perp & \text{iff } \exists \mathbb{D}'_{\varphi,u} \subset \mathbb{D}_{\varphi,u} \text{ s.t.} \\ & \forall D_\varphi \in \mathbb{D}'_{\varphi,u} : [u^{D_\varphi} \models_4 \hat{\varphi}(D_\varphi)] \neq \perp \wedge \\ & \forall D_\varphi \notin \mathbb{D}'_{\varphi,u} : [u^{D_\varphi} \models_4 \hat{\varphi}(D_\varphi)] = \perp \wedge \\ & |\mathbb{D}'_{\varphi,u}| \not\sim c \text{ if } \mathcal{Q} = \mathbb{E} \text{ else } |\mathbb{D}'_{\varphi,u}|/|\mathbb{D}_{\varphi,u}| \not\sim c \wedge \\ & \forall v \in \Sigma^\omega : uv \not\models_4 \varphi \\ \top_p & \text{iff } \exists \mathbb{D}'_{\varphi,u} \subset \mathbb{D}_{\varphi,u} \text{ s.t.} \\ & \forall D_\varphi \in \mathbb{D}'_{\varphi,u} : [u^{D_\varphi} \models_4 \hat{\varphi}(D_\varphi)] \in \{\top, \top_p\} \wedge \\ & \forall D_\varphi \notin \mathbb{D}'_{\varphi,u} : [u^{D_\varphi} \models_4 \hat{\varphi}(D_\varphi)] \notin \{\top, \top_p\} \wedge \\ & |\mathbb{D}'_{\varphi,u}| \sim c \text{ if } \mathcal{Q} = \mathbb{E} \text{ else } |\mathbb{D}'_{\varphi,u}|/|\mathbb{D}_{\varphi,u}| \sim c \wedge \\ & \exists v \in \Sigma^\omega : uv \not\models_4 \varphi \\ \perp_p & \text{iff } \exists \mathbb{D}'_{\varphi,u} \subset \mathbb{D}_{\varphi,u} \text{ s.t.} \\ & \forall D_\varphi \in \mathbb{D}'_{\varphi,u} : [u^{D_\varphi} \models_4 \hat{\varphi}(D_\varphi)] \in \{\top, \top_p\} \wedge \\ & \forall D_\varphi \notin \mathbb{D}'_{\varphi,u} : [u^{D_\varphi} \models_4 \hat{\varphi}(D_\varphi)] \notin \{\top, \top_p\} \wedge \\ & |\mathbb{D}'_{\varphi,u}| \not\sim c \text{ if } \mathcal{Q} = \mathbb{E} \text{ else } |\mathbb{D}'_{\varphi,u}|/|\mathbb{D}_{\varphi,u}| \not\sim c \wedge \\ & \exists v \in \Sigma^\omega : uv \models_4 \varphi \end{cases} \blacksquare$$

The semantics are defined for five cases:

- If φ is an LTL property, then we use the four-valued semantics in RV-LTL.
- Let $\mathbb{D}'_{\varphi,u}$ be a subset that contains all values of the quantified variable that satisfy the inner property. If the cardinality of this subset satisfies the numerical constraint on the quantifier, and no infinite extension of the trace can violate it, the valuation is \top .
- Now, let $\mathbb{D}'_{\varphi,u}$ contain all values for which the inner property is not \perp , i.e. it could be \top , \top_p , or \perp_p . If the cardinality of this subset violates the numerical constraint on the quantifier, and no infinite extension of the trace can satisfy it, the valuation is \perp .
- \top_p is similar to \top , except that $\mathbb{D}'_{\varphi,u}$ can include values with which the inner property evaluates to \top_p , and there exists an extension to the trace prefix that can violate the quantifier constraint.
- \perp_p is the opposite of \top_p , where $\mathbb{D}'_{\varphi,u}$ violates the quantifier constraint, and there exists an extension to the trace prefix that can satisfy the constraint.

Note that $\text{LTL}_4\text{-C}$ semantics are defined recursively from the outermost quantifier. The recursion can be observed in $[u^{D_\varphi} \models_4 \hat{\varphi}(D_\varphi)]$ where D_φ is a

value vector $\langle d \rangle$ for the quantified variable in \mathcal{Q} , and $\hat{\varphi}(D_\varphi)$ is property φ without quantifier \mathcal{Q} . Hence, the semantics recurse with one less quantifier at each step until there are no counting quantifiers and φ is an LTL property, at which case we use RV-LTL semantics. Also note that for a finite prefix of a trace, the semantics of LTL₄-C is decidable since the quantification is over a finite set of objects that exist in the trace.

3 Divide-and-Conquer-based Monitoring of LTL₄-C

In this section, we describe our technique inspired by divide-and-conquer for evaluating LTL₄-C properties at run time. This approach forms the basis of our parallel verification algorithm in Section 4.

Unlike runtime verification of propositional RV-LTL properties, where the structure of a monitor is determined solely based on the property itself, a monitor for an LTL₄-C needs to evolve at run time, since the valuation of quantified variables change over time. More specifically, the monitor \mathcal{M}_φ for an LTL₄-C property $\varphi = \mathcal{Q}_\varphi\psi$ relies on instantiating a *submonitor* for each property instance $\hat{\varphi}$ obtained at run time. We incorporate two type of submonitors: (1) RV-LTL *submonitors* evaluate the inner LTL property ψ . An RV-LTL submonitor instance is denoted as $\mathcal{M}_{D_\varphi}^*$, where D_φ is the unique value vector that binds all quantified variables in the property, leaving only a simple LTL property to be monitored. (2) The second time of submonitors is *quantifier submonitors*, described in Subsection 3.1. In Subsection 3.2, we explain the conditions under which a submonitor is instantiated at run time. Finally, in Subsection 3.3, we elaborate on how submonitors evaluate an LTL₄-C property.

3.1 Quantifier Submonitors

Given a finite trace u and an LTL₄-C property $\varphi = \mathcal{Q}_\varphi\psi$, a *quantifier submonitor* ($\mathcal{M}^\mathcal{Q}$) is a monitor responsible for determining the valuation of a property instance $\hat{\varphi}(D_\varphi|^i)$ with respect to a trace subsequence $u^{D_\varphi|^i}$, if $i < |\mathcal{Q}_\varphi|$.

Definition 5 (Quantifier Submonitor). *Let $\varphi = \mathcal{Q}_\varphi\psi$ be an LTL₄-C property and $\hat{\varphi}(D_\varphi|^i)$ be a property instance, with $i \in [0, |\mathcal{Q}_\varphi| - 1]$. The quantifier submonitor for $\hat{\varphi}(D_\varphi|^i)$ is the tuple $\mathcal{M}_{D_\varphi|^i}^\mathcal{Q} = \langle \mathcal{Q}_i, \mathbb{M}_{D_\varphi|^i}, \mathcal{F} \rangle$, where*

- \mathcal{Q}_i encapsulates the quantifier information (see Subsection 2.4)
- $\mathbb{M}_{D_\varphi|^i}$ is the set of child submonitors (submonitors of child property instances) defined as follows:

$$\mathbb{M}_{D_\varphi|^i} = \begin{cases} \{\mathcal{M}_{D_\varphi'}^* \mid D_\varphi'|^i = D_\varphi|^i\} & \text{if } i = |\mathcal{Q}_\varphi| - 1 \\ \{\mathcal{M}_{D_\varphi'|^{i+1}}^\mathcal{Q} \mid D_\varphi'|^i = D_\varphi|^i\} & \text{if } i < |\mathcal{Q}_\varphi| - 1 \end{cases}$$

- \mathcal{F} is a function that applies the quantifier constraint \mathcal{Q}_i on the truth values of all the child submonitors $\mathbb{M}_{D_\varphi|^i}$.

Thus, if $i = |\mathcal{Q}_\varphi| - 1$, all child submonitors are RV-LTL submonitors. Otherwise, they are quantifier submonitors of the respective child property instances. ■

3.2 Instantiating Submonitors

Let an LTL_4-C monitor \mathcal{M}_φ for property φ evaluate the property with respect to a finite trace $u = u_0u_1 \dots$. Let $D_\varphi = \langle d_0, d_1, \dots \rangle$ be a value vector and u_0 the first trace event such that $\forall d_i : p_i(d_i) \in u_0$. In this case, the LTL_4-C monitor instantiates submonitors for every property instance resulting from that value vector. A value vector of length $|\mathbb{Q}_\varphi|$ results in $|\mathbb{Q}_\varphi|+1$ property instances: one for each quantifier in addition to an RV-LTL inner property. Figure 1 demonstrates the tree structure of submonitors graphically.

3.3 Evaluating LTL4-C Properties

Once the LTL_4-C monitor instantiates its submonitors, every submonitor is responsible for updating its truth value. The truth value of an RV-LTL submonitor (\mathcal{M}^*) is determined by its automaton. Quantifier submonitors update their truth value by applying the quantifier constraint on their child submonitors and producing a valuation based on LTL_4-C semantics.

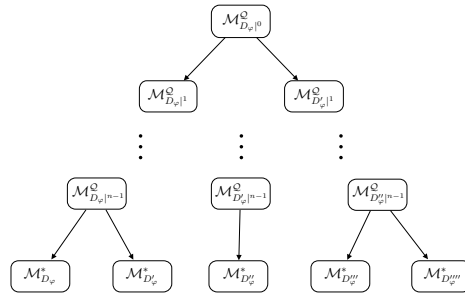


Fig. 1: Tree structure of an LTL_4-C monitor.

4 Parallel RV Algorithm

The main challenge in designing a runtime monitor is to ensure that its behavior does not intervene with functional and extra-functional (e.g., timing constraints) behavior of the program under scrutiny. This section presents a parallel algorithm for verification of LTL_4-C properties. Our idea is that such a parallel algorithm enables us to offload the monitoring tasks into a different computing unit (e.g., the GPU). The algorithm utilizes the *MapReduce* programming model to spawn and merge submonitors to determine the final verdict. It is important to note that the algorithm supports both online and offline monitoring. We generalize the input to the algorithm as a trace, which could be the entire program trace in the case of offline monitoring, or an event or a buffered sequence of events in the case of online monitoring.

This section is organized as follows: Subsection 4.1 describes how valuations are extracted from a trace in run time, and Subsection 4.2 describes the steps of the algorithm in detail.

4.1 Valuation Extraction

Valuation extraction refers to obtaining a valuation of quantified variables from the trace. As described in LTL_4-C semantics, the predicate $p_i(x_i)$ identifies the

subset of the domain of x_i over which the quantifier is applied: namely the subset that exists in the trace. From a theoretical perspective, we check whether the predicate is a member of some trace event, which is a set of predicates. From an implementation perspective, the trace event is a key-value structure, where the key is for instance a string identifying the quantified variable, and the value is the concrete value of the quantified variable in that trace event.

4.2 Algorithm Steps

Algorithm 1 presents the pseudocode of the parallel monitoring algorithm. Given an LTL₄-C property $\varphi = \mathbb{Q}_\varphi \psi$, the input to the algorithm is the RV-LTL monitor \mathcal{M}^* of RV-LTL property ψ , a finite trace u , the set of quantifiers \mathbb{Q}_φ , and the vector of keys K used to extract valuations. Note that the algorithm supports both online and offline runtime verification. Offline mode is straightforward since the algorithm receives a finite trace that it can evaluate.

In the case of online mode, the algorithm maintains data structures that represent the tree structure shown in Figure 1, and repeated invocation of the monitor updates these data structures incrementally. Thus, an online monitor will receive batches of events in run time and process them, building the tree of monitors with every invocation of the monitor. These invocations can be periodic or event based, and the batches can be of any length.

The entry point to the algorithm is at Line 5 which is invoked when the monitor receives a trace to process. This can be the entire trace in offline monitoring, or a buffered segment of the trace in online monitoring. The algorithm returns a truth value of the property at Line 8. Subsections 4.2 – 4.2 describe the functional calls between Lines 5 – 8.

The MapReduce operations are visible in functions *SortTrace* and *ApplyQuantifiers*, which perform a *map* (\mapsto) in Lines 10 and 40 respectively. *ApplyQuantifiers* also performs a *reduction* (\mapsto) in Line 41.

Trace Sorting As shown in Algorithm 1, the first step in the algorithm is to sort the input trace u (Line 5). The function *SortTrace* performs this functionality as follows:

1. The function performs a parallel map of every trace event to the value vector that it holds (Line 10).
2. The mapped trace is sorted in parallel using the quantifier variable as a key (Line 11).
3. The sorted trace is then compacted based on valuations, and the function returns a map μ where keys are value vectors and values are the ranges of where these value vectors exist in trace u (Line 12). A range contains the start and end index. This essentially defines the subsequences u^{D_φ} for each property instance $\hat{\varphi}(D_\varphi)$ (refer to Subsection 2.4).

Monitor Spawning Monitor spawning is the second step of the algorithm (Line 6). The function *SpawnMonitors* receives a map μ and searches the cached collection of previously encountered value vectors \mathbb{D} for duplicates. If a value vector in μ is new, it creates submonitors and inserts them in the tree of submonitors T (Line 18). The function *AddToTree* attempts to generate $|\mathbb{Q}_\varphi| - 1$ quantifier submonitors $\mathcal{M}^\mathbb{Q}$ (Line 22) ensuring there are no duplicate monitors in the tree (Line 23).

After all quantifier submonitors are created, *SpawnMonitors* creates an RV-LTL submonitor \mathcal{M}^* and adds it as a child to the leaf quantifier submonitor in the tree representing the value vector (Line 19). This resembles the structure in Figure 1.

Checking whether submonitors do not already exist and the creation of new submonitors is performed in parallel for all value vectors in trace u . This is because the trace has been sorted and grouped by unique value vectors in the previous step. Thus, each subtree of monitors that corresponds to a unique value vector is created in parallel, and connected to its parent via locks.

Algorithm 1 LTL₄-C Monitor

```

1: INPUT: An RV-LTL monitor  $\mathcal{M}^*$  of LTL property  $\psi$ , a
  finite trace  $u$ , a set of quantifiers  $\mathbb{Q}_\varphi$ , and a vector of
  keys  $K$  to extract valuations of quantified variables.
2: declare  $T = \{\mathcal{M}_{D|0}^\mathbb{Q}\}$   $\triangleright$  Tree of quantifier submonitors
3: declare  $\mathbb{D} = \{\}$ ,  $\triangleright$  Value vector set
4: declare  $\mathbb{M}^* = \{\}$   $\triangleright$  RV-LTL submonitor set
5:  $\mu \leftarrow \text{SORTTRACE}(u)$   $\triangleright$  The entry point
6:  $\text{SPAWNMONITORS}(\mu)$ 
7:  $\text{DISTRIBUTE}(u, \mu)$ 
8: return  $\text{APPLYQUANTIFIERS}(|\mathbb{Q}_\varphi| - 1)$ 



---


9: function  $\text{SORTTRACE}(u)$   $\triangleright$  Trace sorting and compaction
10:  $u_i \Rightarrow u'_i := \text{VALUEVEC}(u_i, K)$   $\triangleright$   $\parallel$  map to value
    vectors
11:  $\text{PARALLELSORT}(u', K)$ 
12:  $\mu(D, r) \leftarrow \text{PARALLELCOMPACT}(u')$ 
13: return  $\mu$ 



---


14: function  $\text{SPAWNMONITORS}(\mu)$   $\triangleright$  Monitor spawning
15: for  $D \in \mu$  do in parallel
16:   if  $D \notin \mathbb{D}$  then
17:      $\text{ADD}(\mathbb{D}, D)$ 
18:      $t \leftarrow \text{ADDTOTREE}(D)$ 
19:      $t.\text{addMonitor}(\text{CREATEMONITOR}(D))$ 



---


20: function  $\text{ADDTOTREE}(D)$ 
21:  $t = T.\text{root}$ 
22: for  $i \in [1, |\mathbb{Q}_\varphi| - 1]$  do
23:   if  $\mathcal{M}_{D|i}^\mathbb{Q} \notin t.\text{children}$  then
24:      $t.\text{addchild}(\mathcal{M}_{D|i}^\mathbb{Q})$ 
25:      $t \leftarrow t.\text{children}[\mathcal{M}_{D|i}^\mathbb{Q}]$ 
26: return  $t$ 



---


27: function  $\text{CREATEMONITOR}(D)$   $\triangleright$  Monitor creation
28:  $\mathcal{M}_D^* \leftarrow \text{LAUNCHMONITORTHREAD}(D)$ 
29:  $\mathcal{M}_D^*.D \leftarrow D$ 
30:  $\text{ADD}(\mathbb{M}^*, \mathcal{M}_D^*)$ 
31: return  $\mathcal{M}_D^*$ 



---


32: function  $\text{DISTRIBUTE}(u, \mu)$   $\triangleright$  Distribute trace to
    monitors
33: for  $\mathcal{M}_D^* \in \mathbb{M}^*$  do in parallel
34:    $\text{PROCESSBUFFER}(\mathcal{M}_D^*, u, \mu[\mathcal{M}_D^*.D])$ 



---


35: function  $\text{PROCESSBUFFER}(\mathcal{M}_D^*, u, r)$   $\triangleright$  Process trace
36: filter include  $u \Rightarrow u' := u[r.\text{start}, r.\text{end}]$   $\triangleright$   $\parallel$  filter
37:  $\mathcal{M}_D^*.b \leftarrow \text{UPDATEMONITOR}(\mathcal{M}_D^*, u')$ 



---


38: function  $\text{APPLYQUANTIFIERS}(i)$   $\triangleright$  Apply quantifiers
39: for  $t \in T.\text{nodesAtDepth}(i)$  do in parallel
40:    $t.\text{children} \Rightarrow \{s := [v, v', \dots]\}$   $\triangleright$   $\parallel$  map
41:    $s \mapsto t.v$   $\triangleright$   $\parallel$  reduction to truth vector
42:    $t.b \leftarrow \text{VALUATION}(t)$   $\triangleright$  LTL4-C semantics
43: if  $i = 0$  then
44:   return  $t.b$ 
45: return  $\text{APPLYQUANTIFIERS}(i - 1)$ 

```

Distributing the Trace The next step in the algorithm is to distribute the sorted trace to all RV-LTL submonitors (Line 7). The function *Distribute* instructs every RV-LTL submonitor to process its respective trace by passing the full trace and the range of its respective subsequence, which is provided by the

map μ (Line 34). The RV-LTL monitor updates its state according to the trace subsequence and stores its truth value b .

Applying Quantifiers Applying quantifiers is a recursive process, beginning with the leaf quantifier submonitors and proceeding upwards towards the root of the tree (Line 8). Function *ApplyQuantifiers* operates in the following steps:

1. The function retrieves all quantifier submonitors at the i^{th} level in the tree T (Line 39).
2. In parallel, for each quantifier submonitor, all child submonitor truth values are reduced into a single truth value of that quantifier submonitor (Lines 40-42). This step *reduces* all child truth vectors into a single vector and then applies LTL_4-C semantics to determine the truth value of the current submonitor, essentially applying function \mathcal{F} on the truth values of all child submonitors.
3. The function proceeds recursively calling itself on submonitors that are one level higher. It terminates when the root of the tree is reached, where the truth value is the final verdict of the property with respect to the trace.

5 Implementation and Experimental Results

We have implemented Algorithm 1 for two computing technologies: Multi-core CPUs and GPUs. We applied three optimizations in our GPU-based implementation: (1) we use *CUDA Thrust API* to implement parallel sort, (2) we use *Zero-Copy Memory* which parallelizes data transfer with kernel operation without caching, and (3) we enforced alignment, which enables coalesced read of trace events into monitor instances. In order to intercept systems calls, we have integrated our algorithm with the Linux `strace` application, which logs all system calls made by a process, including the parameters passed, the return value, the time the call was made, etc. Notice that using `strace` has the benefit of eliminating static analysis for instrumentation.

Subsection 5.1 presents the case studies implemented to study the effectiveness of the GPU implementation in online and offline monitoring. Subsection 5.2 discusses the experimental setup, while Subsection 5.3 analyzes the results.

5.1 Case studies

We have conducted three case studies, the first demonstrates using our implementation for online monitoring, while case study 2 and 3 are monitored offline. Following is a detailed description of the case studies:

1. **Ensuring every request on a socket is responded to.** This case study monitors the responsiveness of a web server. Web servers under heavy load may experience some timeouts, which results in requests that are not responded to. This is a factor contributing to the uptime of the server, along

with other factors like power failure, or system failure. Thus, we monitor that at least 95% of requests are indeed responded to:

$$\mathbb{A}_{\geq 0.95} s : \text{socket}(s) \cdot (\mathbf{G} \text{ receive}(s) \Rightarrow \mathbf{F} \text{ respond}(s))$$

We use the Apache Benchmarking tool to create varying loads on the Apache web server, and monitor the property *online*.

2. **Ensuring fairness in utilization of personal cloud storage services.** This case study is based on the work in [12], which discusses how profiling DropBox traffic can identify the bottlenecks and improve the performance. Among the issues detected during this analysis, is a user repeatedly uploading chunks of maximum size to DropBox servers. Thus, it is beneficial for a runtime verification system to ensure that the majority of chunks are not of maximum size, ensuring fairness of service use. The corresponding LTL_4-C property is as follows:

$$\mathbb{A}u : \text{user}(u) \cdot \mathbb{A}_{< 0.5} c : \text{chunk}(c) \cdot \text{ismaxchunksize}(u, c)$$

3. **Ensuring proxy cache is functioning correctly.** This experiment is based on a study that shows the effectiveness of utilizing proxy cache in decreasing YouTube videos requests in a large university campus [21]. Thus, we monitor that no video is requested externally more than once:

$$\mathbb{A}v : \text{vid}(v) \cdot \mathbb{E}_{\leq 1} r : \text{req}(r, v) \cdot \text{external}(r, v)$$

Notice that the formulas in the above case studies utilize counting semantics to express properties that cannot be expressed in standard LTL. Moreover, evaluation of these properties can never result in permanent satisfaction nor permanent violation (Case Study 1 and 2 only). Thus, the use of four-valued logic allows the monitor to produce a meaningful verdict for these properties.

5.2 Experimental Setup

Experiment Platform. The experiments machine comprises of a 12-core Intel Xeon E5-1650 CPU, an Nvidia Tesla K20c GPU, and 32GB of RAM, running Ubuntu 12.04.

Experimental Factors. We experiment with three implementations: single CPU, parallel CPU, and a GPU based implementation. We also experiment with multiple trace sizes to demonstrate scalability.

Experimental Metrics. We measure the total execution time and the monitor’s CPU utilization. This is to demonstrate the impact of monitoring on overall CPU utilization. We perform 20 replicates of each experiment and present error bars of a 95% confidence interval.

5.3 Results

First of all, the case studies have been evaluated with real-world datasets, except in Case Study 1, where we use the Apache benchmark tool to generate traces. We have validated that the monitor works correctly by ensuring that verdicts of presumably true/false are produced appropriately for Case Studies 1 and 2, and verdicts of presumably true / permanently false are produced appropriately for Case Study 3.

The performance results of Case Study 1 are shown in Figure 2a. As seen in the figure, the GPU implementation scales efficiently with increasing trace size, resulting in the lowest monitoring time of all three implementations. The GPU versus single core CPU speedup ranges from 0.8 to 1.6, increasing with the increasing trace size. When compared to parallel CPU (CPU ||), the speedup ranges from 0.78 to 1.59. This indicates that parallel CPU outperforms GPU for smaller traces (32768), yet does not scale as well as GPU in this case study. This is attributed to the low number of individual objects in the trace, making parallelism less impactful.

CPU utilization results in Figure 2a show a common trend with the increase of trace size. When the trace size is small, parallel implementations incur high CPU utilization as opposed to a single core implementation, which could be attributed to the overhead of parallelization relative to the small trace size. On the other hand, GPU shows a stable utilization percentage, with a 78% average utilization. The single core CPU implementation shows a similar trend, yet slightly elevated average utilization (average 86%). The parallel CPU implementation imposes a higher CPU utilization (average 115%), since more cores are being used to process the trace. This result indicates that shipping the monitoring workload to GPU consistently provides more time for CPU to execute other processes including the monitored process. The results of Case Study 2 and Case Study 3 in Figures 2b and 2c respectively demonstrate a more prominent advantage of using GPU in terms of speedup. The number of individual objects in these traces are large, making parallelism highly effective. For Case Study 2, the speedup of the GPU implementation over single core CPU ranges from 1.8 to 3.6, and 0.83 to 1.18 over parallel CPU. The average CPU utilization of GPU, single core CPU, and parallel CPU is 64%, 82%, and 598% respectively. For Case

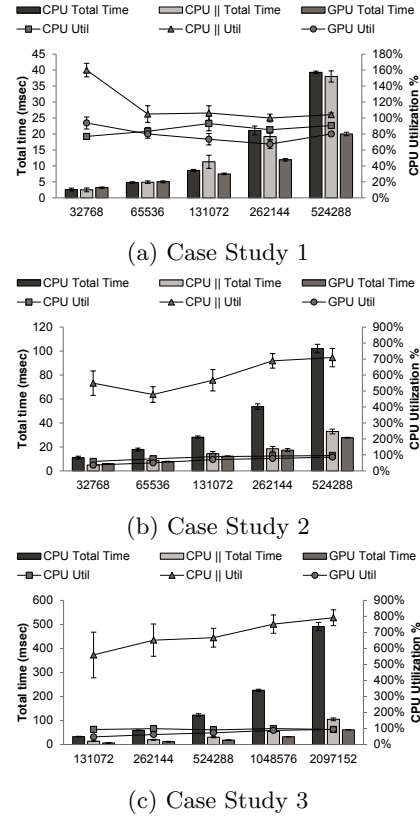


Fig. 2: Experimental results.

Study 3, speedup is more significant, with 6.3 average speedup of GPU over single core CPU, and 1.75 over parallel CPU. The average CPU utilization of GPU, single core CPU, and parallel CPU is 73%, 95%, and 680% respectively. Thus, the parallel CPU implementation is showing large speedup similar to the GPU implementation, yet also results in a commensurate CPU utilization percentage, since most cores of the system are fully utilized.

Although the parallel CPU implementation provides reasonable speedup and the single-core CPU implementation imposes low CPU overhead, the GPU implementation manages to achieve both simultaneously.

6 Related Work

Runtime verification of parametric properties has been studied by Rosu et al [13, 17]. In this line of work, it is possible to build a runtime monitor parameterized by objects in a Java program. The work by Chen and Rosu [9] presents a method of monitoring parametric properties in which a trace is divided into slices, such that each monitor operates on its slice. This resembles our method of identifying trace subsequences and how they are processed by submonitors. However, parametric monitoring does not provide a formalization of applying existential and numerically constrained quantifiers over objects.

Bauer et al. [5] present a formalization of a variant of first order logic combined with LTL. This work is related to our work in that it instantiates monitors at run time according to valuations, and defines quantification over a finite subset of the quantified domain, normally with that subset being defined by the trace. Our work extends this notion with numerical constraints over quantifiers, as well as a parallel algorithm for monitoring such properties. The work by Leucker et al. presents a generic approach for monitoring modulo theories [11]. This work provides a more expressive specification language. Our work enforces a canonical syntax which is not required in [11], resulting in more expressiveness. However, the monitoring solution provided requires SMT solving at run time. LTL_4-C extends RV-LTL by redefining \top_p and \perp_p to support quantifiers and their numerical constraints. This four-valued semantics provides a more accurate assessment of the satisfaction of the property based on finite traces as opposed to the three-valued semantics in [11].

The work in [14] presents an extension to LTL that allows counting events associated with the *Until* operator. In this work, it is possible to apply a numerical constraint on the number of events satisfying subformulas. This differs from LTL_4-C , where numerical constraints are applied on quantified objects, allowing us to reason about the number or percentage of objects that satisfy a property. The work in [2] allows a limited form of quantification over values of a variable, yet does not support a higher level of quantification on the entire LTL property parameterized by the quantified variable, as is possible in LTL_4-C . The work in [19] presents a property specification language that allows quantification, and separates propositional evaluation from quantifier evaluation, similar

to LTL_4-C . However, LTL_4-C supports LTL operators and quantification with numerical constraints.

The work in [1] presents a method of using MapReduce to evaluate LTL properties. The algorithm is capable of processing arbitrary fragments of the trace in parallel. The work in [3] presents a MapReduce method for offline verification of LTL properties with first-order quantifiers. Our approach supports both offline and online monitoring by extending RV-LTL’s four valued semantics, which are capable of reasoning about the satisfaction of a partial trace. This is unclear in [3], since there is no evidence of supporting online monitoring.

The work in [10] presents a specification language for defining properties on input streams. The work in [4] presents an extension to metric first order temporal logic that allows aggregate operations.

Finally, the work in [7, 8] presents two parallel algorithms for verification of propositional LTL specifications at run time. These algorithms are implemented in the tool RiTHM [18]. This paper enhances the framework in [7, 8, 18] by introducing a significantly more expressive formal specification language along with a parallel runtime verification system.

7 Conclusion

In this paper, we proposed a specification language (LTL_4-C) for runtime verification of properties of types of objects in software and networked systems. Our language is an extension of LTL that adds counting semantics with numerical constraints. The four truth values of the semantics of LTL_4-C allows system designers to obtain informative verdicts about the status of system properties at run time. We also introduced an efficient and effective parallel algorithm with two implementations on multi-core CPU and GPU technologies. The results of our experiments on real-world case studies show that runtime monitoring using GPU provides us with the best throughput and CPU utilization, resulting in minimal intervention in the normal operation of the system under inspection.

For future work, we are planning to design a framework for monitoring LTL_4-C properties in distributed systems and cloud services. Another direction is to extend LTL_4-C such that it allows non-canonical strings of quantifiers. Finally, we are currently integrating LTL_4-C in our tool RiTHM [18].

8 Acknowledgement

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Appendix A 4-Valued LTL [6]

First, we note that the syntax of RV-LTL can be easily obtained from Definition 2 by (1) removing the counting quantifier rules and (2) reducing the arity of predicates to 0 (i.e., predicates become atomic propositions).

A.1 FLTL

To introduce RV-LTL semantics, we first introduce Finite LTL. Finite LTL (FLTL) [16] allows us to reason about finite traces for verifying properties at run time. The semantics of FLTL is based on the truth values $\mathbb{B}_2 = \{\top, \perp\}$.

Definition 6 (FLTL semantics). *Let φ and ψ be LTL properties, and $u = u_0u_1 \cdots u_{n-1}$ be a finite trace.*

$$[u \models_{\mathbf{F}} \mathbf{X} \varphi] = \begin{cases} [u_1 \models_{\mathbf{F}} \varphi] & \text{if } u_1 \neq \epsilon \\ \perp & \text{otherwise} \end{cases}$$

$$[u \models_{\mathbf{F}} \varphi \mathbf{U} \psi] = \begin{cases} \top & \exists k \in [0, n) : [u_k \models_{\mathbf{F}} \psi] = \top \wedge \forall l \in [0, k) : [u_l \models_{\mathbf{F}} \varphi] = \top \\ \perp & \text{otherwise} \end{cases}$$

where ϵ is the empty trace. The semantics of FLTL for atomic propositions and Boolean combinations are identical to that of LTL. ■

Similar to standard LTL, $\mathbf{F}p \equiv \top \mathbf{U} p$ and $\mathbf{G}p \equiv \neg \mathbf{F} \neg p$.

A.2 LTL4 Semantics

RV-LTL is designed for runtime verification by producing more informative verdicts than FLTL. The semantics of RV-LTL is defined based on values $\mathbb{B}_4 = \{\top, \top_p, \perp_p, \perp\}$ (*true, presumably true, presumably false, and false* respectively). The semantics of RV-LTL is defined based on the semantics LTL and FLTL.

Definition 7 (RV-LTL semantics). *Let φ be an RV-LTL property and u be a finite prefix of a trace.*

$$[u \models_4 \varphi] = \begin{cases} \top & \forall v \in \Sigma^\omega : uv \models \varphi \\ \perp & \forall v \in \Sigma^\omega : uv \not\models \varphi \\ \top_p & [u \models_{\mathbf{F}} \varphi] \wedge \exists v \in \Sigma^\omega : uv \not\models \varphi \\ \perp_p & [u \not\models_{\mathbf{F}} \varphi] \wedge \exists v \in \Sigma^\omega : uv \models \varphi \blacksquare \end{cases}$$

In this definition, \models denotes the satisfaction relation defined by standard LTL semantics over infinite traces. Thus, an RV-LTL property evaluates to \top with respect to a finite trace u , if the property remains *permanently satisfied*, meaning that for all possible infinite continuations of the trace, the property will always be satisfied in LTL. Likewise, a valuation of \perp means that the property will be *permanently violated*. If the property evaluates to \top_p , this denotes that currently the property is satisfied yet there exists a continuation that could violate it. Finally, value \perp_p denotes that currently the property is violated yet there exists a continuation that could satisfy it.

LTL4 Monitors In [6], the authors introduce a method of synthesizing a *monitor*, as a deterministic finite state machine (FSM), for an RV-LTL property.

Definition 8 (RV-LTL Monitor). Let φ be an RV-LTL formula over Σ . The monitor \mathcal{M}_φ of φ is the FSM $(\Sigma, Q, q_0, \delta, \lambda)$, where Q is a set of states, q_0 is the initial state, $\delta \subseteq Q \times \Sigma \times Q$ is the transition relation, and $\lambda : Q \rightarrow \mathbb{B}_4$ is a function such that:

$$[u \models_4 \varphi] = \lambda(\delta(q_0, u)). \blacksquare$$

Thus, given an RV-LTL property φ and a finite trace u , monitor \mathcal{M}_φ is capable of producing a truth value in \mathbb{B}_4 , which is equal to $[u \models_4 \varphi]$. For example, Figure 3 shows the monitor for property $\varphi = \mathbf{G}a \vee (b \mathbf{U} c)$. Observe that a monitor has two *trap* states (only an outgoing self loop), which map to truth values \top and \perp . They are trap states since these truth values imply permanent satisfaction (respectively, violation). Otherwise, states labeled by \top_p and \perp_p can have outgoing transitions to other states.

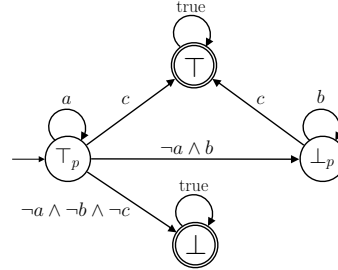


Fig. 3: RV-LTL monitor for $\varphi = \mathbf{G}a \vee (b \mathbf{U} c)$.

Appendix B LTL₄—C Example

Property 3 illustrates an example of an \mathbb{E} quantifier that can be permanently violated. Also, since the \mathbb{A} quantifier in Property 3 defaults to $\mathbb{A}_{=1}$, it will be violated if a single user makes more than three unauthorized login attempts. In such a case, the entire property evaluates to \perp . Table 1 illustrates how permanent satisfaction or violation apply to the different numerical constraints of \mathbb{E} quantifiers.

$$\varphi = \mathbb{A}x : \text{user}(x) \cdot (\mathbb{E}_{\leq 3} r : \text{rid}(r) \cdot (\text{login}(r, x) \wedge \text{unauthorized}(r, x))) \quad (3)$$

To clarify the semantics, consider Property 3 and the following program trace:

```
{rid(12), user(Adam), login(12, Adam), unauthorized(12, Adam)}
{rid(13), user(Adam), login(13, Adam), unauthorized(13, Adam)}
{rid(14), user(Jack), login(14, Jack), authorized(14, Jack)}
{rid(15), user(Adam), login(15, Adam), unauthorized(15, Adam)}
{rid(16), user(Adam), login(16, Adam), unauthorized(16, Adam)}
```

where each line represents an event: a set of interpreted predicates. Each event contains a request identifier (rid), a username, a request type (login), and response

status (authorized or unauthorized). As seen in the trace, there are 5 distinct value vectors making up the set $\mathbb{D}_{\varphi,u}$:

$$\mathbb{D}_{\varphi,u} = \{\langle Adam, 12 \rangle, \langle Adam, 13 \rangle, \langle Jack, 14 \rangle, \langle Adam, 15 \rangle, \langle Adam, 16 \rangle\}$$

Now, let us apply the inductive semantics on the property.

Table 1: Rules of permanent satisfaction or violation of \mathbb{E} constraints

Operator	Verdict
$> c$	Permanent satisfaction if $> c$
$\geq c$	Permanent satisfaction if $\geq c$
$= c$	Permanent violation if $> c$
$< c$	Permanent violation if $\geq c$
$\leq c$	Permanent violation if $> c$

Step 1. We begin by checking the truth value of $[u \models_4 \varphi]$, which requires determining which condition in Definition 4 applies. Note that in Definition 4 $\varphi = \mathcal{Q}\psi$, and hence we are only concerned with the outer \mathbb{A} quantifier at this point.

Since φ is not an LTL property, the first condition of Definition 4 does not apply. To determine the truth value, we observe $\mathbb{D}_{\varphi,u}$. Since the property only has one quantifier at this step, $\mathbb{D}_{\varphi,u} = \{Adam, Jack\}$. As shown in the definition, we are trying to determine the valuation of $[u^{D_\varphi} \models_4 \hat{\varphi}(D_\varphi)]$ for every $D_\varphi \in \mathbb{D}_{\varphi,u}$. For every user in $\mathbb{D}_{\varphi,u}$, we are trying to check whether u^{D_φ} models $\hat{\varphi}(D_\varphi)$ in LTL_4-C .

- u^{D_φ} is the trace subsequence that contains events concerned with that user (where $\text{user}(D_\varphi)$ holds). For instance,

$$\begin{aligned} u^{Adam} = & \{\text{rid}(12), \text{user}(Adam), \text{login}(12, Adam), \text{unauthorized}(12, Adam)\}, \\ & \{\text{rid}(13), \text{user}(Adam), \text{login}(13, Adam), \text{unauthorized}(13, Adam)\}, \\ & \{\text{rid}(15), \text{user}(Adam), \text{login}(15, Adam), \text{unauthorized}(15, Adam)\}, \\ & \{\text{rid}(16), \text{user}(Adam), \text{login}(16, Adam), \text{unauthorized}(16, Adam)\} \end{aligned}$$

- $\hat{\varphi}(D_\varphi)$ at this point is the property φ excluding the first quantifier and binding x to D_φ . For instance:

$$\hat{\varphi}(Adam) = \mathbb{E}_{\leq 3} r : \text{rid}(r) \cdot (\text{login}(r, Adam) \wedge \text{unauthorized}(r, Adam))$$

The next step is to determine the valuation of the two substraces u^{Adam} and u^{Jack} , which brings us back recursively to Definition 4.

Step 2. We start by evaluating $[u^{Adam} \models_4 \hat{\varphi}(Adam)]$. In this case, $\mathbb{D}_{\varphi,u} = \{12, 13, 15, 16\}$, and thus we need to evaluate $[u^{D_\varphi} \models_4 \hat{\varphi}(D_\varphi)]$ for every request ID. For instance,

$$u^{12} = \{\text{rid}(12), \text{user}(Adam), \text{login}(12, Adam), \text{unauthorized}(12, Adam)\}$$

Similarly,

$$\hat{\varphi}(12) = \text{login}(12, \text{Adam}) \wedge \text{unauthorized}(12, \text{Adam})$$

The next step is to determine the valuation of the subtraces $u^{12}, u^{13}, u^{15}, u^{16}$ against their respective $\hat{\varphi}$, which brings us back recursively to Definition 4.

Step 3. Since the properties are now standard LTL properties, we use RV-LTL to determine their valuation. The following are the valuations of all the LTL properties:

$$[u^{\langle \text{Adam}, 12 \rangle} \models_4 \hat{\varphi}(\langle \text{Adam}, 12 \rangle)] = \top$$

$$[u^{\langle \text{Adam}, 13 \rangle} \models_4 \hat{\varphi}(\langle \text{Adam}, 13 \rangle)] = \top$$

$$[u^{\langle \text{Adam}, 15 \rangle} \models_4 \hat{\varphi}(\langle \text{Adam}, 15 \rangle)] = \top$$

$$[u^{\langle \text{Adam}, 16 \rangle} \models_4 \hat{\varphi}(\langle \text{Adam}, 16 \rangle)] = \top$$

Next, we bubble back up to the evaluation of $[u^{\text{Adam}} \models_4 \hat{\varphi}(\text{Adam})]$.

Step 4. After evaluating the inner property we are now back to evaluating $[u^{\text{Adam}} \models_4 \hat{\varphi}(\text{Adam})]$. We now have valuations for every $D_\varphi \in \mathbb{D}_{\varphi, u} = \{12, 13, 15, 16\}$. Upon observing the semantics in Definition 4, we can see that the valuation is \perp , since there exists a subset $\mathbb{D}'_{\varphi, u} = \{12, 13, 15, 16\}$ for which every value satisfies the property (see above), and the numerical constraint on the $\mathbb{E}_{\leq 3}$ quantifier is violated ($4 > 3$). Since no future continuation of this trace can satisfy the property, this property is permanently violated.

Step 5. We now repeat steps 2 – 4 for user *Jack*, essentially evaluating $[u^{\text{Jack}} \models_4 \hat{\varphi}(\text{Jack})]$. In this case, $\mathbb{D}_{\varphi, u} = \{14\}$, and $[u^{14} \models_4 \hat{\varphi}(14)] = \perp$. The valuation of $[u^{\text{Jack}} \models_4 \hat{\varphi}(\text{Jack})]$ is \top_p since there exists a subset $\mathbb{D}'_{\varphi, u} = \{\}$ whose cardinality satisfies the numerical constraint on the $\mathbb{E}_{\leq 3}$ quantifier ($0 < 3$). However, there is a continuation of the trace that violates the property, namely receiving four unauthorized login attempts for user *Jack*. Hence, the valuation is \top_p .

Step 6. We now go back to evaluating the outer quantifier we attempted to evaluate in Step 1. Recall that $\mathbb{D}_{\varphi, u} = \{\text{Adam}, \text{Jack}\}$. We now know that

$$[u^{\text{Adam}} \models_4 \hat{\varphi}(\text{Adam})] = \perp$$

$$[u^{\text{Jack}} \models_4 \hat{\varphi}(\text{Jack})] = \top_p$$

Upon applying the semantics in Definition 4, we reach the conclusion that the property is permanently violated. Let us try to deduce that by checking the conditions for every truth value in Definition 4:

- Permanently true (\top): The subset $\mathbb{D}'_{\varphi, u} = \{\}$ is the only subset that satisfies the first two conditions of the conjunction. However, the cardinality of the subset violates the numerical constraint ($\frac{0}{2} \neq 1$), and hence the valuation is *not* permanently true.

- Presumably true (\top_p): The subset $\mathbb{D}'_{\varphi,u} = \{Jack\}$ satisfies the two conditions of the conjunction. However, the cardinality of the subset violates the numerical constraint ($\frac{1}{2} \neq 1$). Hence the valuation is *not* presumably true.
- Presumably false (\perp_p): The subset $\mathbb{D}'_{\varphi,u} = \{Jack\}$ satisfies the two conditions of the conjunction. The cardinality of the subset violates the numerical constraint ($\frac{1}{2} \neq 1$). However, there does not exist a continuation of the trace that would satisfy the numerical constraint, since it requires 100% satisfaction. Hence, the valuation is *not* presumably false.
- Permanently false (\perp): The subset $\mathbb{D}'_{\varphi,u} = \{Jack\}$ is the only subset that satisfies the first two conditions of the conjunction. The cardinality of the subset violates the numerical constraint ($\frac{1}{2} \neq 1$), and any continuation of the trace will continue to violate the property. Hence the valuation is **permanently false**.