User’s Guide to Editing, Typesetting and Type-Checking Z Specifications with the ESZ Toolkit\textsuperscript{1}

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\textsuperscript{1}For Version 1.1b of the type checker, with small corrections for the new version 2.x. See augmenting documents in the distribution for the new version 2.x.

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Abstract

This manual describes the usage of the ESZ-toolkit for the editing, typesetting and type-checking of Z specifications. The ESZ-toolkit provides an implementation of the Z notation which conforms to the Z reference manual [?], basing on a \LaTeX\-representation which is downwards-compatible to the wide-spread \textsc{fUZZ} tool [?]. It implements some minor language extensions compared to \textsc{fUZZ}, namely position-independence of declarations, automatic inclusion of sub-documents and \LaTeX\-based mixfix operator-symbols.

The \textit{ESZ}-toolkit currently consists of the following components:

- The \textit{ESZ}{}-type-checker analyzes Z specifications and produces diagnostics regarding syntax and context errors.
- The \textit{ESZ}{}-\LaTeX{}-package supports the type-setting of specifications.
- The \textit{ESZ}{}-emacs-mode supports the editing of Z specifications as an add-on to the editing-mode AUCTeX.

Furthermore, prototypical support for the Espress notation \textit{$\mu$SZ} is included.

\textbf{Note:} This manual is incomplete. Currently, only the most important differences w.r.t. other type checkers are documented. Tutorial material is mostly missing.
Contents

1 Typesetting Z Specifications .......................................................... 2
  1.1 Environments ................................................................. 2
  1.2 Symbols ................................................................. 3
  1.3 Controlling the Layout .................................................. 3

2 Type-Checking Z Specifications ......................................................... 6
  2.1 Running the Type-Checker ................................................ 6
  2.2 The Z language supported by the Type-Checker ..................... 6
  2.3 Directives ................................................................. 7
    2.3.1 Specifying the Toolkit ........................................... 7
    2.3.2 Specifying Symbol Fixity ....................................... 7
    2.3.3 Other Directives ................................................ 8

3 The Emacs Environment ............................................................... 9

A Unit-Weighted Rational Numbers .................................................... 10
  A.1 Concepts ................................................................. 10
  A.2 Symbols ................................................................. 10
  A.3 Specification .......................................................... 13
Chapter 1

Typesetting Z Specifications

This chapter describes how to typeset Z specifications with the \textsc{ESZ} package which comes with \textsc{ESZ}. The typesetting support is enabled by including the following command in the header section of the \LaTeX package:

\begin{verbatim}
\usepackage[options]{esz}
\end{verbatim}

Several package options may be specified to control the behaviour of the \textsc{ESZ} package; these will be explained as we proceed.

1.1 Environments

Here are the environments which embedd Z specifications:

\begin{verbatim}
\begin{zed} items \end{zed}
\begin{syntax} items \end{syntax}
\begin{schema} {Name}[generics] decls \where property \end{schema}
\begin{schema*} [generics] decls \where property \end{schema*}
\begin{axdef} decls \where property \end{axdef}
\begin{gendef} [generics] decls \where property \end{gendef}
\end{verbatim}
1.2 Symbols

Table 1.1 contains the separators and keywords used in Z specifications. Table 1.2 on the next page shows the operators from the language base as well as the mathematical toolkit as defined by the ZRM [?]. In the right column of this table the syntactical class of the operator is given.

If you have a license for using the Z font oxsz which comes with the fUZZ distribution, you may instruct the \texttt{ESZ} package to use this font with the packet-option \texttt{oxsz}:

\begin{verbatim}
\usepackage[...oxsz,...]{esz}
\end{verbatim}

The \texttt{oxsz} font produces slightly nicer arrows and other symbols. (This manual is type-set with the \texttt{oxsz} font.)

1.3 Controlling the Layout

An explicit line-break is created with the command $\backslash \backslash$. Alternatively, the command $\texttt{\backslash also}$ may be used to create a line-break with some extra vertical space.
<table>
<thead>
<tr>
<th>( Z )</th>
<th>( \text{\texttt{\textbackslash num}} )</th>
<th>( N )</th>
<th>( \text{\texttt{\textbackslash nat}} )</th>
<th>\texttt{word}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \emptyset )</td>
<td>( \text{\texttt{\textbackslash emptyset}} )</td>
<td>( \cap )</td>
<td>( \text{\texttt{\textbackslash setminus}} )</td>
<td>\texttt{inop 1}</td>
</tr>
<tr>
<td>( \mapsto )</td>
<td>( \text{\texttt{\textbackslash mapsto}} )</td>
<td>( ; )</td>
<td>( \text{\texttt{\textbackslash comp}} )</td>
<td>\texttt{inop 2}</td>
</tr>
<tr>
<td>( \upto )</td>
<td>( \text{\texttt{\textbackslash upto}} )</td>
<td>( | )</td>
<td>( \text{\texttt{\textbackslash setminus}} )</td>
<td>\texttt{inop 3}</td>
</tr>
<tr>
<td>( + )</td>
<td>( \text{\texttt{\textbackslash plus}} )</td>
<td>( \div )</td>
<td>( \text{\texttt{\textbackslash div}} )</td>
<td>\texttt{inop 4}</td>
</tr>
<tr>
<td>( \setminus )</td>
<td>( \cup )</td>
<td>( \text{\texttt{\textbackslash setminus}} )</td>
<td>\texttt{inop 5}</td>
<td></td>
</tr>
<tr>
<td>( \text{\texttt{\textbackslash uminus}} )</td>
<td>( \text{\texttt{\textbackslash uminus}} )</td>
<td>( \text{\texttt{\textbackslash uminus}} )</td>
<td>\texttt{inop 6}</td>
<td></td>
</tr>
<tr>
<td>( \ast )</td>
<td>( \text{\texttt{\textbackslash star}} )</td>
<td>( \text{\texttt{\textbackslash star}} )</td>
<td>\texttt{postop}</td>
<td></td>
</tr>
<tr>
<td>( \equiv )</td>
<td>( \text{\texttt{\textbackslash \in}} )</td>
<td>( \in )</td>
<td>\texttt{inrel}</td>
<td></td>
</tr>
<tr>
<td>( \neq )</td>
<td>( \text{\texttt{\textbackslash neq}} )</td>
<td>( \text{\texttt{\textbackslash neq}} )</td>
<td>\texttt{inrel}</td>
<td></td>
</tr>
<tr>
<td>( \subseteq )</td>
<td>( \text{\texttt{\textbackslash subseteq}} )</td>
<td>( \text{\texttt{\textbackslash subseteq}} )</td>
<td>\texttt{inrel}</td>
<td></td>
</tr>
<tr>
<td>( \langle )</td>
<td>( \leq )</td>
<td>( \text{\texttt{\textbackslash leq}} )</td>
<td>\texttt{inrel}</td>
<td></td>
</tr>
<tr>
<td>( \rangle )</td>
<td>( \geq )</td>
<td>( \text{\texttt{\textbackslash geq}} )</td>
<td>\texttt{inrel}</td>
<td></td>
</tr>
<tr>
<td>( \inbag )</td>
<td>( \text{\texttt{\textbackslash \inbag}} )</td>
<td>( \text{\texttt{\textbackslash \inbag}} )</td>
<td>\texttt{inbag}</td>
<td></td>
</tr>
<tr>
<td>( \text{\texttt{\textbackslash \prefix}} )</td>
<td>( \text{\texttt{\textbackslash \prefix}} )</td>
<td>( \text{\texttt{\textbackslash \prefix}} )</td>
<td>\texttt{prefix}</td>
<td></td>
</tr>
<tr>
<td>( \text{\texttt{\textbackslash \suffix}} )</td>
<td>( \text{\texttt{\textbackslash \suffix}} )</td>
<td>( \text{\texttt{\textbackslash \suffix}} )</td>
<td>\texttt{suffix}</td>
<td></td>
</tr>
<tr>
<td>( \text{\texttt{\textbackslash \disjoint}} )</td>
<td>( \text{\texttt{\textbackslash \disjoint}} )</td>
<td>( \text{\texttt{\textbackslash \disjoint}} )</td>
<td>\texttt{disjoint}</td>
<td></td>
</tr>
<tr>
<td>( \leftrightarrow )</td>
<td>( \\text{\texttt{\textbackslash \rel}} )</td>
<td>( \leftrightarrow )</td>
<td>\texttt{\rel}</td>
<td></td>
</tr>
<tr>
<td>( \rightarrow )</td>
<td>( \\text{\texttt{\textbackslash \fun}} )</td>
<td>( \rightarrow )</td>
<td>\texttt{\fun}</td>
<td></td>
</tr>
<tr>
<td>( \mapsto )</td>
<td>( \\text{\texttt{\textbackslash \inj}} )</td>
<td>( \rightarrow )</td>
<td>\texttt{\inj}</td>
<td></td>
</tr>
<tr>
<td>( \mapsto )</td>
<td>( \\text{\texttt{\textbackslash \surj}} )</td>
<td>( \rightarrow )</td>
<td>\texttt{\surj}</td>
<td></td>
</tr>
<tr>
<td>( \mapsto )</td>
<td>( \\text{\texttt{\textbackslash \ffun}} )</td>
<td>( \rightarrow )</td>
<td>\texttt{\ffun}</td>
<td></td>
</tr>
<tr>
<td>( \text{\texttt{\textbackslash \power}} )</td>
<td>( \text{\texttt{\textbackslash \power}} )</td>
<td>( \text{\texttt{\textbackslash \power}} )</td>
<td>\texttt{\power}</td>
<td></td>
</tr>
<tr>
<td>( \text{\texttt{\textbackslash \id}} )</td>
<td>( \text{\texttt{\textbackslash \id}} )</td>
<td>( \text{\texttt{\textbackslash \id}} )</td>
<td>\texttt{id}</td>
<td></td>
</tr>
<tr>
<td>( \text{\texttt{\textbackslash \iseq}} )</td>
<td>( \text{\texttt{\textbackslash \iseq}} )</td>
<td>( \text{\texttt{\textbackslash \iseq}} )</td>
<td>\texttt{iseq}</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.2: Operators
Normally, the contents of boxed $Z$ environments are kept on a single page. The command `\zbreak` may be used to specify an optional page-break; it behaves like `\also` if no page-break is necessary.

Horizontal space is inserted by the command `\`, as in $f^*x$, which typesets as $fx$ (instead of $\tilde{fx}$, where no tilde has been used). The amount of space inserted by `\` can be controlled by the command `\esztildespace`:

```
\esztildespace{spacingcommand}
```

`spacingcommand` may be any valid spacing command in math mode such as `\;`. The default is `\.`.

Absolute indentation may be created with the `\tn` command, where $n$ is a number in range 1 to 9. Here is an example how to use absolute indentation:

```
\begin{axdef}
\ldots
\begin{axdef}
\ldots
cache' =
if \#(dom cache) = limit
then
(cache \ldots)
\cup \{key \mapsto data\}
else
\begin{axdef}
\ldots
\end{axdef}
\begin{axdef}
\ldots
cache \cup \{key \mapsto data\}
```

Relativ indentation in the style of the `program` package is created by the commands `\<` and `\>`. The command `\<` sets the future left margin to the current horizontal position and the command `\>` resets it to the original value. These commands may be nested. Here is how the above example might be typeset with relative indentation:

```
\begin{axdef}
\ldots
\begin{axdef}
\ldots
\begin{axdef}
\ldots
\end{axdef}
\end{axdef}
```

Note that it is necessary to close all opened margins at the end of an environment.
Chapter 2

Type-Checking Z Specifications

This chapter describes how to type-check Z specifications with ESZ.

2.1 Running the Type-Checker

The type-checker is normally called from within the ESZ emacs-mode (c.f. Chapter 3 on page 9).

For the moment, see the UNIX manual page esz(1) for a description how to manually call the type-checker.

2.2 The Z language supported by the Type-Checker

The ESZ type-checker supports a language which is downwards compatible to the fUZZ-language and confirms to the ZRM [?]. The following extensions compared to fUZZ have been added:

- the textual order of declarations and of directives is not relevant for ESZ. The only restriction is that declarations of constants, variables and of named schemes must be acyclic.

- Subdocuments included with the LaTeX command \input will be also included by the type checker.

- A special syntax for mixfix operators, defined by a parameterized TeX-command is supported (see Section 2.3.2).

- Repeated declarations of global constants are allowed provided that they are type-compatible. The behavior is similar as with repeated declarations of variables in schemes.

- The mathematical toolkit to be used may be specified by a directive (see Section 2.3.1 on the next page).

[more to come]
2.3 Directives

A directive is a kind of declaration which effects the behaviour of the type-checker but is not part of the Z language. ESZ supports all directives known from fUZZ plus some additional ones which are specific for ESZ.

A directive must always start on the beginning of a line, introduced with the phrase \texttt{%\%keyword}, where \texttt{keyword} defines the kind of the directive.

2.3.1 Specifying the Toolkit

Starting with version 1.1, the mathematical toolkit to be used for type-checking is specified with a directive:

\texttt{%\%toolkit "toolkitname"}

Note that a toolkit is not loaded by default, you have to put the \texttt{%\%toolkit} somewhere in the document (preferable in the document header).

The \texttt{toolkitname} refers to the name of a file which contains legal ESZ input. The file is searched for in the current directory and in the ESZ library directory. The library directory is taken from the UNIX environment variable \texttt{ESZLIB}; if this variable is undefined, a default is taken which is burned into ESZ at installation time.

ESZ currently comes with the following toolkits:

- \texttt{zrm}: this toolkit is compatible to fUZZ resp. the ZRM [?].
- \texttt{zrm+units}: this toolkit is downwards compatible to the fUZZ toolkit, but replaces the arithmetic system by unit-weighted rational numbers (see Appendix A on page 10).

2.3.2 Specifying Symbol Fixity

The type-checker recognizes several directives which specify fixity information for symbols. The following fixity directives are the ones known from fUZZ:

\texttt{%\%inop word ...n}

\texttt{%\%postop word ...}

\texttt{%\%prerel word ...}

\texttt{%\%inrel word ...}

\texttt{%\%pregen word ...}

\footnote{In other languages this concept is also called a \textit{pragma}}

\footnote{The UNIX command line option \texttt{-p} is still supported, but the recommended way is to use the \texttt{%\%toolkit} directive.}
%\%ingen word ...

Here, \( n \) is in range 1..6. See Table 1.2 on page 4 for the default assignment of fixitys to symbols from the ZRM based toolkits.

\LaTeX{}-based mixfix operators, a special feature of \texttt{ESZ}, are declared with the following directives:

\begin{itemize}
  \item \texttt{\%\%texop word \ldots n}
  \item \texttt{\%\%texrel word \ldots n}
  \item \texttt{\%\%texgen word \ldots n}
\end{itemize}

\texttt{word} should be a \LaTeX{} command. \( n \) is the number of \LaTeX{} arguments taken by the command; it must be in range 1..4.

A symbol declared as a \LaTeX{}-based mixfix must always be supplied with \textit{all} arguments. If, for example, the directive

\begin{verbatim}
\%\%texop \cmd 2
\end{verbatim}

has been given, then one writes

\begin{verbatim}
\cmd{\Exp1}{\Exp2}
\end{verbatim}

to apply \texttt{\cmd} in an expression, which is equivalent to

\begin{verbatim}
(\cmd{\_}{\_})(\Exp1,\Exp2)
\end{verbatim}

The form with \texttt{\_} is also used in declarations such as in

\begin{verbatim}
\cmd{\_}{\_} : A \cross B \fun C
\end{verbatim}

\subsection{Other Directives}

[more to come]
Chapter 3

The Emacs Environment

For the moment, see the document “The ESZ Mode – a minor emacs mode extending AUC \TeX for Z documents” which comes with \textit{ESZ}.
Appendix A

Unit-Weighted Rational Numbers

This chapter describes a ZRM based toolkit which extends the arithmetic system by so-called unit-weighted rational numbers\(^1\). To use this toolkit, the option \texttt{units} must be supplied to the \texttt{ESZ} \LaTeX{}-package and the toolkit \texttt{zrm+units} to the type-checker:

\begin{verbatim}
\usepackage[...\texttt{units}]{esz}
...
\%\texttt{toolkit "zrm+units"}
\end{verbatim}

A.1 Concepts

Unit-weighted rational numbers are rational numbers as known from mathematics which are attributed with a mapping from physical units to an exponent. The number \(\frac{3}{4}\) \(\text{m}\), for example, consists of the rational number \(\frac{3}{4}\) and the mapping \((\lambda u : U \cdot 0) \cup \{\text{Metre} \mapsto 1, \text{Second} \mapsto -2\}\). This mapping assigns the exponent 1 to the unit metre, the exponent -2 to the unit second, and the exponent 0 to all other units \(u\).

Unit-weighted rational numbers are introduced as a given type \(Q_U\). All other numbers – rational numbers, integer numbers, natural numbers – are derived as subsets of \(Q_U\). The usual arithmetic operations are defined on \(Q_U\); some of the operations are partial (e.g. addition of unit-weighted numbers is defined only if the units coincide).

A.2 Symbols

Figure A.1 on the following page shows the symbols newly introduced or redefined by the \texttt{zrm+units} toolkit (in comparison to the standard ZRM toolkit). Figure A.2 on page 12 shows the \LaTeX{} representation of these symbols together with their syntactical class.

\(^1\)It is planned to further extend to real numbers; nearly all to be said about unit-weighted rational numbers will take over to unit-weighted real numbers.
$C4DE +
-t'$

Figure A.1: New or modified Symbols of the zrm+units Toolkit
<table>
<thead>
<tr>
<th>Symbol</th>
<th>LaTeX Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>\unit</td>
<td>\unit</td>
<td>Unit</td>
</tr>
<tr>
<td>Kilogram</td>
<td>Kilogram</td>
<td>Kilogram</td>
</tr>
<tr>
<td>Ampere</td>
<td>Ampere</td>
<td>Ampere</td>
</tr>
<tr>
<td>Candela</td>
<td>Candela</td>
<td>Candela</td>
</tr>
<tr>
<td>LENGTH</td>
<td>LENGTH</td>
<td>LENGTH</td>
</tr>
<tr>
<td>TIME</td>
<td>TIME</td>
<td>TIME</td>
</tr>
<tr>
<td>TEMPERATURE</td>
<td>TEMPERATURE</td>
<td>TEMPERATURE</td>
</tr>
<tr>
<td>LIGHTINTENSITY</td>
<td>LIGHTINTENSITY</td>
<td>LIGHTINTENSITY</td>
</tr>
<tr>
<td>AMOUNT</td>
<td>AMOUNT</td>
<td>AMOUNT</td>
</tr>
<tr>
<td>FORCE</td>
<td>FORCE</td>
<td>FORCE</td>
</tr>
<tr>
<td>ACCELERATION</td>
<td>ACCELERATION</td>
<td>ACCELERATION</td>
</tr>
<tr>
<td>\wmat</td>
<td>\wmat</td>
<td>\wmat</td>
</tr>
<tr>
<td>\num</td>
<td>\num</td>
<td>\num</td>
</tr>
<tr>
<td>\nat</td>
<td>\nat</td>
<td>\nat</td>
</tr>
<tr>
<td>\weight</td>
<td>\weight</td>
<td>\weight</td>
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<tr>
<td>\recipro</td>
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<tr>
<td>\min</td>
<td>\min</td>
<td>\min</td>
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<tr>
<td>\nom</td>
<td>\nom</td>
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<tr>
<td>\floor</td>
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<tr>
<td>\m</td>
<td>\m</td>
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<tr>
<td>\s</td>
<td>\s</td>
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</tr>
<tr>
<td>\kel</td>
<td>\kel</td>
<td>\kel</td>
</tr>
<tr>
<td>\mol</td>
<td>\mol</td>
<td>\mol</td>
</tr>
<tr>
<td>\upto</td>
<td>\upto</td>
<td>\upto</td>
</tr>
<tr>
<td>+</td>
<td>\plus</td>
<td>\plus</td>
</tr>
<tr>
<td>\star</td>
<td>\star</td>
<td>\star</td>
</tr>
<tr>
<td>\mod</td>
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<tr>
<td>$</td>
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<tr>
<td>&lt;</td>
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<td>&lt;</td>
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<tr>
<td>&gt;</td>
<td>&gt;</td>
<td>&gt;</td>
</tr>
<tr>
<td>\frac</td>
<td>\frac</td>
<td>\frac</td>
</tr>
</tbody>
</table>

Figure A.2: LaTeX representation and syntactical class of zrm+units Symbols
A.3 Specification

For the moment, here are just the specifications from [?]; see there for explanations.

\[ [U, Q_U] \]

**Metre, Kilogram, Second, Ampere, Kelvin, Candela, Mole : U**

Metre \( \neq \) Kilogram
Metre \( \neq \) Second
\(...\)
Kilogram \( \neq \) Second
Kilogram \( \neq \) Ampere
\(...\)

\[ Q : P \forall U \]
weight : \( Q_U \rightarrow U \rightarrow Q \)
\[ Q = \{ x : Q_U \mid \text{weight} \ x = (\lambda \ u : U \bullet 0) \} \]

\[ Z, N, N_1 : P \forall Q \]
0, 1, 2, \ldots : N

\[ N = (\lambda \ a : Q \bullet a + 1)^* \{ 0 \} \]
\[ Z = N \cup \{ n : N \bullet - n \} \]
\[ N_1 = N \setminus \{ 0 \} \]

\[ \_ \_ \_ : Q_U \times (U \rightarrow Q) \rightarrow Q_U \]
val : \( Q_U \rightarrow Q \)
\[ \forall x : Q_U ; \ x = \text{val} \ x \ \_ \_ \_ \ \text{weight} \ x \]
\[ \forall a, a' : Q ; \ w, w' : U \rightarrow Q \bullet \]
\[ (a \ \_ \_ \_ \ w = a' \ \_ \_ \_ \ w' \Rightarrow a = a' \land w = w') \land \]
\[ \text{val}(a \ \_ \_ \_ \ w) = a \land \text{weight}(a \ \_ \_ \_ \ w) = w \]

\[ \forall x : Q_U ; \ w, w' : U \rightarrow Q \bullet \]
\[ (x \ \_ \_ \_ \ w) \ \_ \_ \_ \ w' = x \ \_ \_ \_ \ (\lambda \ u : U \bullet w \ u + w' \ u) = x \ \_ \_ \_ \ w \ * 1 \ \_ \_ \_ \ w' \]
\[ \forall x : Q_U ; \ pw : U \rightarrow Q \bullet \]
\[ x \ \_ \_ \_ \ pw = x \ \_ \_ \_ \ ((\lambda \ u : U \bullet 0) \uplus pw) \]
\[
\begin{align*}
\_ + \_ : & \mathcal{Q}_U \times \mathcal{Q}_U \rightarrow \mathcal{Q}_U \\
\_ \cdot \_ : & \mathcal{Q}_U \times \mathcal{Q}_U \rightarrow \mathcal{Q}_U \\
\_ : & \mathcal{Q}_U \rightarrow \mathcal{Q}_U \\
\frac{1}{\_} : & \mathcal{Q}_U \rightarrow \mathcal{Q}_U \\
\_ < \_ : & \mathcal{Q}_U \leftrightarrow \mathcal{Q}_U \\
\end{align*}
\]

\[
\text{dom}(\_ + \_) = \{x, x' : \mathcal{Q}_U \mid \text{weight } x = \text{weight } x'\}
\]

\[
\text{dom} \frac{1}{\_} = \{x : \mathcal{Q}_U \mid \text{val } x \neq 0\}
\]

\[
\forall a, a' : \mathcal{Q}; w : U \rightarrow \mathcal{Q} \bullet
\begin{align*}
\_ \cdot w + a' & \_ \cdot w = (a + a') \_ \cdot w \land \\
-(a \_ \cdot w) & = -a \_ \cdot w
\end{align*}
\]

\[
\forall a, a' : \mathcal{Q}; w, w' : U \rightarrow \mathcal{Q} \bullet
\begin{align*}
a \_ \cdot w \cdot a' \_ \cdot w' & = (a \cdot a') \_ \cdot (\_ \cdot w \cdot w' \cdot u) = ((a \cdot a') \_ \cdot w) \_ \cdot w' \land \\
\frac{1}{\_} (a \_ \cdot w) & = \frac{1}{\_} a \_ \cdot (\_ \cdot w \cdot u)
\end{align*}
\]

\[
\forall a, a' : \mathcal{Q}; w, w' : U \rightarrow \mathcal{Q} \bullet
\begin{align*}
a \_ \cdot w < a' \_ \cdot w' & \iff a < a' \land w = w'
\end{align*}
\]

\[
\begin{align*}
\_ - \_ : & \mathcal{Q}_U \times \mathcal{Q}_U \rightarrow \mathcal{Q}_U \\
\text{abs} : & \mathcal{Q}_U \rightarrow \mathcal{Q}_U \\
\_ \leq & \_ \geq \_ : \mathcal{Q}_U \leftrightarrow \mathcal{Q}_U \\
\end{align*}
\]

\[
\text{dom}(\_ - \_) = \text{dom}(\_ + \_)
\]

\[
\forall x, y : \mathcal{Q} \bullet x - y = x + (-y)
\]

\[
\text{dom}(\_ / \_) = \mathcal{Q}_U \times (\mathcal{Q}_U \setminus \{x : \mathcal{Q}_U \mid \text{val } x = 0\})
\]

\[
\forall x, y : \mathcal{Q} \bullet x \div y = x \cdot (\frac{1}{\_} y)
\]

\[
\forall x, y : \mathcal{Q} \bullet \frac{x}{y} = x \div y
\]

\[
\text{abs} = (\lambda x : \mathcal{Q}_U \bullet \text{if val } x > 0 \text{ then } x \text{ else } -x)
\]

\[
(\_ \leq \_) = (\_ < \_) ^{\sim}
\]

\[
(\_ \leq \_) = (\_ < \_) \cup \text{id } \mathcal{Q}_U
\]

\[
(\_ \geq \_) = (\_ \leq \_) ^{\sim}
\]

\[
\_ - \_ : Z \times Z \rightarrow \mathbb{P} Z
\]

\[
\_ \cdot \_ : \mathcal{Q}_U \times \mathcal{Q}_U \rightarrow \mathbb{P} \mathcal{Q}_U
\]

\[
\text{min}, \text{max} : \mathbb{P} \mathcal{Q}_U \rightarrow \mathcal{Q}_U
\]

\[
\forall n, m : Z \bullet n \ldots m = \{i : Z \mid n \leq i \leq m\}
\]

\[
\text{dom}(\_ \cdot \_) = \{x, y : \mathcal{Q}_U \mid \text{weight } x = \text{weight } y\}
\]

\[
\forall x, y : \mathcal{Q} \bullet x \cdot y = \{z : \mathcal{Q}_U \mid x \leq z \leq y\}
\]

\[
\text{min} = \{Q : \mathbb{P} \_ : \mathcal{Q}_U \mid \exists x', Q \bullet x = x' \land (\forall y : Q \bullet x' \leq y)\}
\]

\[
\text{max} = \{Q : \mathbb{P} \_ : \mathcal{Q}_U \mid \{q : Q \bullet \_ \rightarrow \neg x \rightarrow \min\}\}
\]
\[
\text{\textit{succ}} : \mathbb{N} \rightarrow \mathbb{N}
\]
\[
\text{\textit{floor}}, \text{\textit{ceil}} : \mathbb{Q} \rightarrow \mathbb{Z}
\]
\[
\text{\textit{\_div}} \; , \; \text{\textit{\_mod}} : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}
\]
\[
\text{\textit{nom}}, \text{\textit{dnom}} : \mathbb{Q} \rightarrow \mathbb{Z}
\]
\[
\text{\textit{succ}} = (\lambda n : \mathbb{N} \bullet n + 1)
\]
\[
\forall \ a : \mathbb{Q} \bullet
\]
\[
\text{floor} \ a = \max \{n : \mathbb{Z} \mid n \leq a\} \land
\]
\[
\text{ceil} \ a = -(\text{floor}(-a))
\]
\[
\text{\textit{dom(\_div \_)}} = \text{\textit{dom(\_mod \_)}} = \mathbb{Z} \times (\mathbb{Z} \setminus \{0\})
\]
\[
\forall \ n, m : \mathbb{Z} \bullet
\]
\[
\ text{\textit{n div m}} = \text{\textit{floor(n / m)}} \land
\]
\[
(n \text{\textit{div m}}) \ast m + n \text{\textit{mod m}} = n
\]
\[
\forall \ a : \mathbb{Q} \bullet
\]
\[
\text{\textit{a}} = \text{\textit{nom a / dnom a}} \land
\]
\[
(a = 0 \Rightarrow \text{\textit{nom a}} = 0 \land \text{\textit{dnom a}} = 1) \land
\]
\[
(a \neq 0 \Rightarrow \neg (\exists n : \mathbb{Z} \mid n > 1 \bullet
\]
\[
\text{\textit{nom a mod n}} = 0 \land \text{\textit{dnom a mod n}} = 0)
\]
\[
\text{(\_m)}
\]
\[
\text{\textit{length}} = (\lambda x : \mathbb{Q} \bullet x \text{\textit{\{M}etre \leftrightarrow 1\}})
\]
\[
\text{(\_kg)}
\]
\[
\text{\textit{mass}} = (\lambda x : \mathbb{Q} \bullet x \text{\textit{\{K}ilogram \leftrightarrow 1\}})
\]
\[
\text{(\_s)}
\]
\[
\text{\textit{time}} = (\lambda x : \mathbb{Q} \bullet x \text{\textit{\{S}econd \leftrightarrow 1\}})
\]
\[
\text{(\_amp)}
\]
\[
\text{\textit{current}} = (\lambda x : \mathbb{Q} \bullet x \text{\textit{\{A}mpere \leftrightarrow 1\}})
\]
\[
\text{(\_kel)}
\]
\[
\text{\textit{temperature}} = (\lambda x : \mathbb{Q} \bullet x \text{\textit{\{K}elvin \leftrightarrow 1\}})
\]
\[
\text{(\_can)}
\]
\[
\text{\textit{lightintensity}} = (\lambda x : \mathbb{Q} \bullet x \text{\textit{\{C}andela \leftrightarrow 1\}})
\]
\[
\text{(\_mol)}
\]
\[
\text{\textit{amount}} = (\lambda x : \mathbb{Q} \bullet x \text{\textit{\{M}ole \leftrightarrow 1\}})
\]
\[
\text{\textit{length}} = \{x : \mathbb{Q} \bullet x \text{\textit{\{M}etre \leftrightarrow 1\}}\}
\]
\[
\text{\textit{mass}} = \{x : \mathbb{Q} \bullet x \text{\textit{\{K}ilogram \leftrightarrow 1\}}\}
\]
\[
\text{\textit{time}} = \{x : \mathbb{Q} \bullet x \text{\textit{\{S}econd \leftrightarrow 1\}}\}
\]
\[
\text{\textit{current}} = \{x : \mathbb{Q} \bullet x \text{\textit{\{A}mpere \leftrightarrow 1\}}\}
\]
\[
\text{\textit{temperature}} = \{x : \mathbb{Q} \bullet x \text{\textit{\{K}elvin \leftrightarrow 1\}}\}
\]
\[
\text{\textit{lightintensity}} = \{x : \mathbb{Q} \bullet x \text{\textit{\{C}andela \leftrightarrow 1\}}\}
\]
\[
\text{\textit{amount}} = \{x : \mathbb{Q} \bullet x \text{\textit{\{M}ole \leftrightarrow 1\}}\}
\]
\[
\text{\textit{speed}} = \{x : \mathbb{Q} \bullet x \text{\textit{\{M}etre \leftrightarrow 1, S}econd \leftrightarrow -1\}}\}
\]
\[
\text{\textit{force}} = \{x : \mathbb{Q} \bullet x \text{\textit{\{M}etre \leftrightarrow 1, K}ilogram \leftrightarrow 1, S}econd \leftrightarrow -2\}}\}
\]
\[
\text{\textit{acceleration}} = \{x : \mathbb{Q} \bullet x \text{\textit{\{M}etre \leftrightarrow 1, S}econd \leftrightarrow -2\}}\}}\}
\]