mTrust: Discerning Multi-Faceted Trust in a Connected World

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ABSTRACT

Traditionally, research about trust assumes a single type of trust between users. However, trust, as a social concept, inherently has many facets indicating multiple and heterogeneous trust relationships between users. Due to the presence of a large trust network for an online user, it is necessary to discern multi-faceted trust as there are naturally experts of different types. Our study in product review sites reveals that people place trust differently to different people. Since the widely used adjacency matrix cannot capture multi-faceted trust relationships between users, we propose a novel approach by incorporating these relationships into traditional rating prediction algorithms to reliably estimate their strengths. Our work results in interesting findings such as heterogeneous pairs of reciprocal links. Experimental results on real-world data from Epinions and Ciao show that our work of discerning multi-faceted trust can be applied to improve the performance of tasks such as rating prediction, facet-sensitive ranking, and status theory.

Categories and Subject Descriptors

H.3.3 [**Information Search and Retrieval**]: Information Filetering; H.5.3 [**Group and Organization Interfaces**]: Web-based interaction

General Terms

Algorithms, Experimentation, Human Factors

Keywords

Multi-faceted Trust, Multi-dimension Tie Strength, Trust Network, Heterogeneous Trust

1. INTRODUCTION

In recent years, the notion of trust has attracted more and more attention from the computer science community [6, 18]. Trust plays a central role in exchanging relationships involving unknown risk [4], which provides information about with whom we should share information and from whom we should accept information [5]. The role of trust is especially critical in some online communities such as e-commerce sites and product review sites, which has been described as the "wild wild west" of the 21st century [19].

Users in product review sites such as Epinions[1] can share their reviews about products. Also they can establish their trust networks from which they may seek advice to make decisions. Assuming that users are likely to have similar preferences with their trust networks, trust networks are widely exploited in collaborative filtering [8, 1], intelligent recommender systems [17, 5], review quality prediction [15] and viral marketing [22] to improve the accuracy.

Most of these works assume single and homogeneous trust relationships between users. However, trust, as a social concept, has many facets [5], indicating multiple and heterogeneous trust relationships between users. People's multi-faceted interests and experts of different types suggest that people may place trust differently to different people. Siegler et al. pointed out that there is a strong and significant correlation between trust and similarity [25]. The more similar two people are, the greater the trust between them exists. In the context of product review sites, trust relationships between users can be indicated by their rating similarities [1].

Figure 1(a) demonstrates single trust relationships between a real user from Epinions[2], represented by user 1, and her 20 representative friends. Figures 1(b) and 1(c) show their multi-faceted trust relationships in the categories of "Home & Garden" and "Restaurants", respectively[3]. The width of arcs in these figures indicates their trust strengths. The top 3 trustworthy people in Figures 1(a), 1(b) and 1(c) are users $\{7, 8, 18\}$, $\{19, 8, 6\}$ and $\{7, 9, 11\}$, respectively. They are very different from one another, which reveals the existence of multi-faceted and heterogeneous trust relationships. For instance, user 7 is the most trustworthy person when assuming a single trust relationship. However, he is not even in the top 3 people in "Home & Garden", in which user 19 is most trustable. We closely examine user 19 from Epinions and find that she is the lead reviewer in "Home & Garden". Thus the user should seek advice from user 19 in "Home & Garden" instead of user 7. Moreover, users

[1]http://www.epinions.com
[2]http://www.epinions.com/user-nancy35c
[3]Figure 1 is drawn by Pajek (http://pajek.imfm.si)

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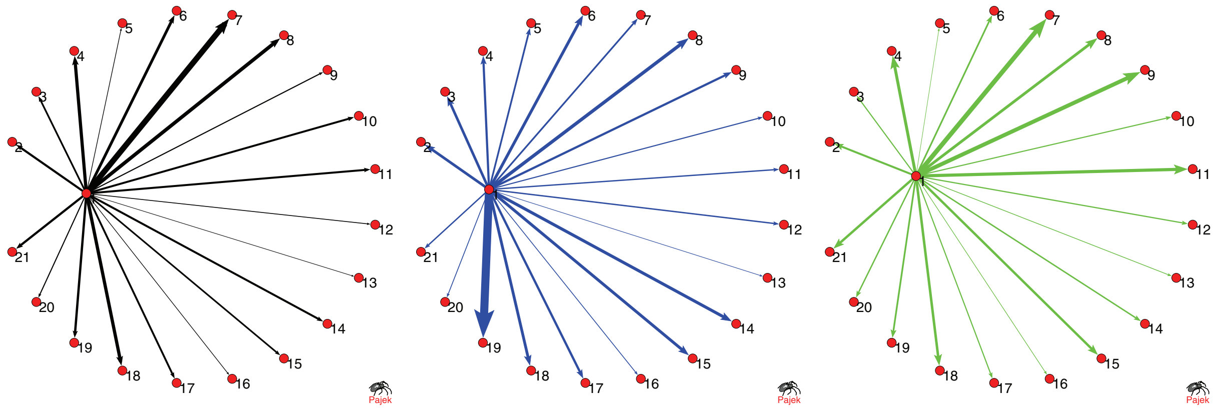


(a) Single Trust (b) Trust in Home & Garden (c) Trust in Restaurants

Figure 1: Single Trust and Multi-Faceted Trust Relationships of One User in Epinions

might trust some people more than others. For example, Figure 1(b) and 1(c) show that their trust strengths vary.

The example from Epinions suggests that single trust can't capture the real relationships between users. To obtain multi-faceted trust relationships between users, there are two challenges: 1) *how to represent multiple and heterogeneous trust relationships between users*? and 2) *how to estimate their strengths*? In this paper, we discern multi-faceted trust relationships in product review sites to improve data-mining related performance. Since an adjacency matrix cannot capture multi-faceted trust relationships, a fine-grained representation is proposed. By incorporating these relationships into rating prediction, their strengths are estimated. Experiments show that our work can improve the performance in various tasks such as rating prediction, facet-sensitive ranking, and status theory. Main contributions of this work include:

- Demonstrating that people with trust relationships have more similar multi-faceted interests than those without and show the existence of multi-faceted trust relationships in product review sites.
- Proposing a fine-gained representation to capture the multi-faceted trust relationships between users. By incorporating these relationships into rating prediction, their strengths are estimated.
- Presenting interesting findings from this multi-faceted trust study. For example, more than 17% of transitive trust relationships are heterogeneous, and more than 23% pairs of reciprocal links are heterogeneous.
- Showing various applications of mTrust such as rating prediction, facet-sensitive ranking and status theory and the experiments using real-world datasets.

The rest of paper is organized as follows. Section 2 describes the datasets used in our work, discusses multi-faceted interests and demonstrates multi-faceted trust relationships between users. Section 3 introduces a fine-grained representation for multi-faceted trust relationships and describes our method to estimate their strengths. Section 4 discusses applications of mTrust. Section 5 presents experimental results and findings. Section 6 briefly reviews related work. Finally, Section 7 concludes this study with future work.

Table 1: Statistics of the Datasets

	Epinions	Ciao
# of Users	22166	12375
# of Products	296277	106797
# of Catagories	27	28
# of Rating	922267	484086
# of Links	355813	237350
Ave Rating	4.05	4.21
Trust Network Density	0.0014	0.0031
Clustering Coefficient	0.1518	0.1969

2. DATASETS AND DATA ANALYSIS

For the purpose of this study, we crawled two datasets from two popular product review sites Epinions and Ciao[4] in the month of May, 2011. On both sites, people not only write critical reviews for various products but also read and rate the reviews written by others. Furthermore, people can add members to their trust networks or "Circle of Trust", if they find their reviews consistently interesting and helpful.

We started with a set of most active users and then did breadth-first search until no new users could be found. For each user, we collected information about profiles, trust networks, and product rating entries. For each product rating entry, we collected date, product name, *categories* of a product and its ratings. Users with fewer than 5 reviews are pruned. Some statistics of the datasets are shown in Table 1: Epinions has a much larger trust network while Ciao has more close-knit trust relationships, indicated by its higher clustering coefficient and network density.

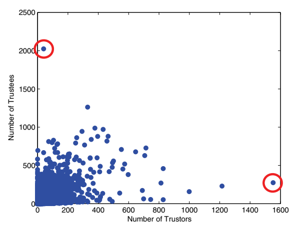
Both sites employ a 5-star rating system. We investigate their rating distributions and find that more than **70%** ratings are 4 or 5. Many studies also reported this positive ratings phenomenon in online consumer ratings [1, 7].

[4]http://www.ciao.co.uk

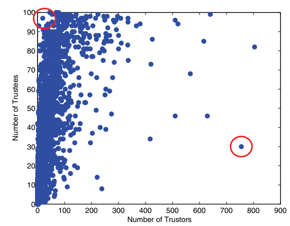
19.8% and **21.3%** of users wrote **80%** reviews in Epinions and Ciao respectively.

2.1 Reciprocity in Trust Relationships

High reciprocity is reported in many relationships such as following relationships in Twitter [23]. We study the reciprocity of trust relationships by showing the correlation between the number of trustors and trustees for each user in Figure 2. The first observation is that most people have few trustors and trustees, while a few users have an extremely high number of trustees or trustors. We also compute the trustors and trustees for each user and the distributions are shown in Figure 3. These distributions suggest a power law distribution that is typical in social networks.



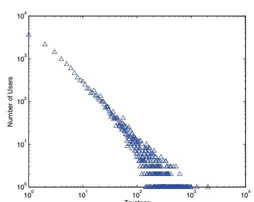
(a) Epinions



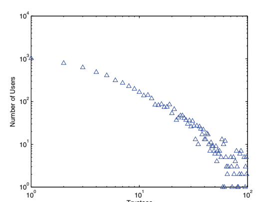
(b) Ciao

Figure 2: Number of Trustors vs Number of Trustees

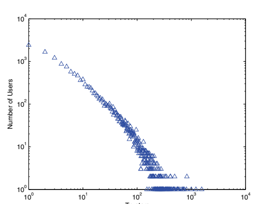
Having many trustors does not necessarily mean having many trustees, and vice versa. Some representative users are indicated by red circles in Figure 2. Our closer examination reveals that the reciprocity is **19.28%** in Epinions and **23.77%** in Ciao.



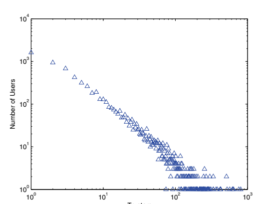
(a) Trustees(Epinions)



(b) Trustees(Ciao)



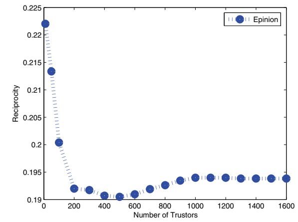
(c) Trustors(Epinions)



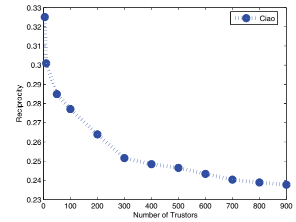
(d) Trustors(Ciao)

Figure 3: Trustors and Trustees Distributions in Epinions and Ciao

We also investigate the correlation between reciprocity and the number of trustors. The results are shown in Figure 4. We find that *people who have fewer trustors are more likely to trust their trustees*. It shows that people with many trustees are more likely to have bias or propensity to trust others [20]. Thus the ratios of reciprocal links for these people are low.



(a) Epinions



(b) Ciao

Figure 4: Reciprocity vs Number of Trustors

2.2 Multi-Faceted Interests

When people place multi-faceted trust to others, they are supposed to have similar multi-faceted interests with their trustees. In this section, we investigate the connection between multi-faceted interests and trust relationships. More specifically, we want to answer this question: *Do people with trust relationships have more similar multi-faceted interests than those without?*. To answer this question, we need to define facet in product review sites and how to measure the multi-faceted interests similarity between a pair of users.

Facet in product review sites is defined as a set of products which are similar to each other. In product review sites, the notion of category is used to organize products, and products in the same category have similar characteristics. Products are manually assigned to different categories by reviewers. Based on these facts, in our work, categories are regarded as facets. On average, people are interested in 6.3 and 5.8 facets in Epinions and Ciao, respectively.

Let $fd_i(k)$ be the probability that user i is interested in facet k, which is formally defined in Eq. (1)

$$fd_i(k) = \frac{n_i(k)}{n_i} \tag{1}$$

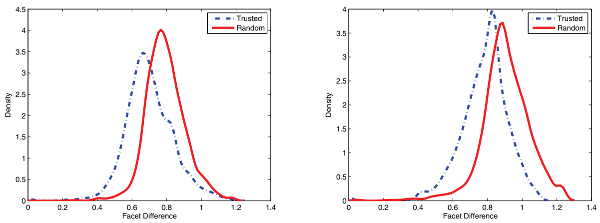
where n_i is the total number of products rated by user i and $n_i(k)$ is the number of products from facet k rated by user i. With the facet distribution for each user, *multi-faceted interests similarity(f_{dist}) between user i and user j can be measured based on the Jensen-Shannon Divergence between two facet distributions fd_i and fd_j.*

$$\begin{aligned} f_{dist} &= \sqrt{2 * D_{JS}(i,j)} \\ &= \sqrt{D_{KL}(fd_i \| m) + D_{KL}(fd_j \| m)} \end{aligned} \tag{2}$$

where m is the average of the two distributions, i.e., $m = \frac{1}{2}(fd_i + fd_j)$. D_{KL} is the Kullback-Leibler Divergence. For example $D_{KL}(fd_i \| m) = \sum_k fd_i(k) \log \frac{fd_i(k)}{m(k)}$

For each user i, we calculate two f_{dist}, i.e., $\mathbf{s}_t(i)$ and $\mathbf{s}_r(i)$. $\mathbf{s}_t(i)$ is the average f_{dist} between user i and his/her trust network, while $\mathbf{s}_r(i)$ is the average f_{dist} between user i and randomly chosen users, who are not in the trust network of user i. The number of these randomly chosen users is the same as the size of user i 's trust network.

For a visual comparison, in Figure 5 we plot the Kernel-smoothing density estimations based on the vectors $\mathbf{s}_t$ and



(a) Epinions (b) Ciao

Figure 5: Density Estimates of Users' *f-dist*

Table 2: Statistics of Chosen Categories

Order	Name	Products	Ratings	Ave Score
Epinions				
1	Electronics	9594	23571	3.96
2	Home & Garden	16029	28759	4.12
3	Computer Hardware	7584	17532	4.02
4	Hotels & Travel	11723	33410	4.15
5	Restaurants	14368	32477	3.91
6	Kids & Family	20926	50783	4.11
Ciao				
Order	Name	Products	Ratings	Ave Score
1	Entertainment	5059	14349	4.13
2	House & Garden	5805	15398	4.37
3	Household Apps	5334	14478	4.34
4	Electronics	4802	12426	4.41
5	Family	6227	18923	4.08
6	Games	6237	18491	4.31

$\mathbf{s}_r$. For both datasets, $\mathbf{s}_t$ have smaller concentrated values, i.e., smaller f_{dist}, compared with $\mathbf{s}_r$.

We also conduct a two sample t-test on the vectors $\mathbf{s}_t$ and $\mathbf{s}_r$. The null hypothesis is H_0: $\mathbf{s}_t = \mathbf{s}_r$, and the alternative hypothesis is H_1: $\mathbf{s}_t < \mathbf{s}_r$. For both datasets, the null hypothesis is rejected at significant level $\alpha = 0.01$ with p-value of 3.2e-7 and 1.3e-6 in Epinions and Ciao, respectively.

The evidence from both Figure 5 and t-test suggests a positive answer to the question: *with high probability, users with trust relationships have smaller f_{dist} than those without*. Based on this finding, we further study multi-faceted relationships in the next section.

2.3 Multi-Faceted Trust Relationships

In order to trust, people need some information usually based on the target person's current and previous experiences [2]. People with similar experiences are more likely to trust each other. In this section, we study rating similarity between users for each facet(*facet similarity*). Due to the strong correlation between user similarity and trust, multi-faceted trust relationships between users can be indicated by facet similarities.

We choose six representative facets (categories) from Epinions and Ciao to study multi-faceted trust relationships. The statistical information for these chosen facets is shown in Table 2. On average, each product receives more than two ratings. The average ratings for different facets are different and users have different preferences for different facets.

For each user, we calculate the average and the variance of similarities with her/his trust network(trust facet similarity) and randomly chosen users(random facet similarity) for each facet. Facet rating similarities between users are calculated by *cosine* similarity. Let $\mathbf{S}_t(i,k)$ and $\mathbf{S}_r(i,k)$ denote the average of trust facet similarities and random facet similarities of user i in facet k, respectively. $\mathbf{V}_t(i,k)$ and $\mathbf{V}_r(i,k)$ are their variances.

Let $\bar{\mathbf{s}}_t(k)$ denote the average of trust facet similarities of all users in facet k, which is formally defined as:

$$\bar{\mathbf{s}}_t(k) = \frac{\sum_{i=1}^{n} \mathbf{S}_t(i,k)}{n} \tag{3}$$

where n is the number of users. Let $\bar{\mathbf{s}}_r$ be the averages of random facet similarities, $\bar{\mathbf{v}}_t$ and $\bar{\mathbf{v}}_r$ be the averages of their variances, respectively. $\bar{\mathbf{s}}_r$, $\bar{\mathbf{v}}_t$ and $\bar{\mathbf{v}}_r$ can be obtained similarly as $\bar{\mathbf{s}}_t$. Table 3 shows the results in the Epinions dataset and we can get similar results in Ciao.

$\bar{\mathbf{s}}_t$ is always larger than $\bar{\mathbf{s}}_r$. This supports that *for each facet, users with trust relationships have more similar product ratings than those without*. $\bar{\mathbf{v}}_t$ is always larger than $\bar{\mathbf{v}}_r$, indicating that *users trust their friends differently and they have greater trust in some friends than in others*. The variance of facet similarity directly supports that the strengths of trust relationships vary between users.

Most of the time, the facet similarities are smaller than the overall similarity, while the facet variances are always larger than the overall variance. In other words, *for each facet, people only strongly trust a part of their trust networks*, or the existence of multi-faceted trust between users.

We also want to investigate whether distributions of facet similarities are significantly different. Thus for each pair of facets (i,j), we conduct a two-sample Kolmogorov-Smirnov(KS) test on their facet similarity vectors, i.e. $\{\mathbf{S}_t(:,i), \mathbf{S}_t(:,j)\}$. The p-value is shown in Table 4. The star next to the p-value means that there is strong evidence($p < 0.01$) that two samples come from different distributions.

The table shows that p-values for all pairs of facets are close to zero. This implies that the distributions for different facets are significantly different, indicating multi-faceted trust relationships in product review sites.

Evidence from Table 3 and KS-test suggests the existence of multi-faceted relationships between users in product review sites. In the next section, we present our method, mTrust, to model multi-faceted relationships, including a fine-grained representation and the estimation of their strengths.

3. MODELING mTrust RELATIONSHIPS

Before building the mathematical model, we would like to establish the notations that are used. Following the standard notations, scalars are denoted by low-case letters $(a, b, \ldots)$, vectors are written as low-case bold letters $(\mathbf{a}, \mathbf{b}, \ldots)$, matri-

Table 3: Average of Means and Variances of Facet Similarities in Epinions

Facets	Trust Network		Random	
	$\bar{\mathbf{s}}_t$	$\bar{\mathbf{v}}_t$	$\bar{\mathbf{s}}_r$	$\bar{\mathbf{v}}_r$
1	0.0128	0.0024	0.0056	0.0011
2	0.0067	0.0016	0.0025	0.0007
3	0.0051	0.0017	0.0016	0.0007
4	0.0053	0.0018	0.0018	0.0007
5	0.0150	0.0024	0.0064	0.0012
6	0.0171	0.0024	0.0077	0.0011
overall	0.0159	0.0017	0.0056	0.0005

Table 4: Statistics of Facet Similarity For Pairs of Facets

Epinions(p-value)					
	2	3	4	5	6
1	5.51e-46*	1.77e-111*	5.14e-76*	2.35e-08*	2.15e-26*
2	-	7.67e-37*	3.65e-16*	2.21e-77*	5.56e-135*
3	-	-	7.91e-06*	4.39e-122*	2.36e-227*
4	-	-	-	3.90e-97*	1.83e-178*
5	-	-	-	-	7.88e-22*

Ciao(p-value)					
	2	3	4	5	6
1	6.00e-24*	6.46e-97*	3.50e-69*	4.63e-7*	4.15e-20*
2	-	1.12e-33*	8.23e-16*	4.72e-26*	1.50e-80*
3	-	-	9.56e-05*	1.42e-92*	3.55e-184*
4	-	-	-	1.49e-69*	1.42e-154*
5	-	-	-	-	1.03e-19*

ces correspond to bold-face captitals($\mathbf{A}, \mathbf{B}, \ldots$), and tensors are written as calligraphic letters ($\mathcal{A}, \mathcal{B}, \ldots$). Also we represent the elements in a given above structure using a convention similar to *Matlab*. For example, $\mathbf{A}(i,j)$ is the entry at the i^{th} row and j^{th} column of the matrix $\mathbf{A}$, $\mathbf{A}(i,:)$ is the i^{th} row of $\mathbf{A}$ and $\mathbf{A}(:,j)$ is the j^{th} column of $\mathbf{A}$ etc.

Let $\mathbf{u} = \{u_1, u_2, \ldots, u_n\}$ be the user set vector where n is the number of users. $\mathbf{e} = \{e_1, e_2, \ldots, e_m\}$ and $\mathbf{f} = \{f_1, f_2, \ldots, f_K\}$ denote the set of products and facets, respectively, where m is the number of products and K is the number of facets. We use $\mathbf{A} \in \mathbb{R}^{n \times n}$ to denote the adjacency matrix for the trust network, in which $\mathbf{A}(i,j) = 1$ if u_j trusts u_i and $\mathbf{A}(i,j) = 0$ otherwise. $\mathbf{R} \in \mathbb{R}^{n \times m}$ denotes the rating matrix and $\mathbf{R}(i,j)$ is the rating for the product e_j from u_i. $\mathbf{PF} \in \mathbb{R}^{m \times K}$ is the product-facet matrix. $\mathbf{PF}(i,k) = 1$ if e_i belongs to f_k and $\mathbf{PF}(i,k) = 0$, otherwise.

3.1 Tensor Representation for Multi-Faceted Trust Relationships

A trust network is often represented by an adjacency matrix with binary values. Multi-faceted trust between users is a **quadruple** {*user, user, facet, strength*}, which is out of the scope of what an adjacency matrix can capture. Thus we extend the *matrix* representation to a *tensor* representation by adding an extra dimension *facets*, as demonstrated in Figure 6.

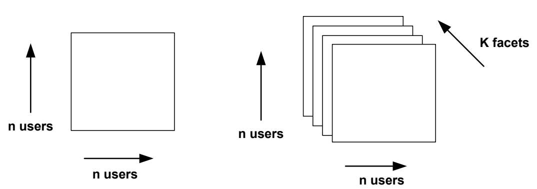


Figure 6: Matrix Representation(Left) vs Tensor Representation(Right)

We first briefly introduce some background on tensors. A *tensor*, also known as multidimensional matrix [3], is a higher order generalization of a vector (first order tensor) and a matrix (second order tensor). An Nth-order tensor $\mathcal{A}$ is denoted as $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N}$. A generalization of the product of two matrices is the product of a tensor and a matrix. The mode-n product of a tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N}$ and a matrix $\mathbf{Q} \in \mathbb{R}^{J_n \times I_n}$ is a tensor, denoted as

$$\mathcal{A} \times_n \mathbf{Q} \in \mathbb{R}^{I_1 \times \ldots \times I_{n-1} \times J_n \times I_{n+1} \ldots I_N} \tag{4}$$

whose entries are given by

$$(\mathcal{A} \times_n \mathbf{Q})(i_1, \ldots, i_{n-1}, j_n, i_{n+1}, \ldots, i_N) = \sum_{i_n} \mathcal{A}(i_1, \ldots, i_{n-1}, i_n, i_{n+1}, \ldots, i_N)\mathbf{Q}(j_n, i_n) \tag{5}$$

The proposed representation, as shown in Figure 6 (**Right**), is a *third order* tensor. Let $\mathcal{A} \in \mathbb{R}^{n \times n \times K}$ represent multi-faceted trust relationships among n users in K facets. Each element $\mathcal{A}(i,j,k)$ indicates the strength that u_j trusts u_i in facet f_k. In the next section, we introduce a method to estimate the strengths of multi-faceted trust relationships, i.e., entities in $\mathcal{A}$.

3.2 Estimation of Strengths of Multi-faceted Trust Relationships

Due to the strong correlation between rating similarity and trust, it is desirable to understand strengths of a user's multi-faceted trust relations in the context of product rating prediction. The (u,i) pairs for which $\mathbf{R}(u,i)$ is known are stored in the set $\mathcal{O} = \{(u,i)|\mathbf{R}(u,i) \textit{ is known}\}$ and unknown pairs are stored in $\mathcal{U} = \{(u,i)|\mathbf{R}(u,i) \textit{ is unknown}\}$. We denote the predicted value of $\mathbf{R}(u,i)$ as $\hat{\mathbf{R}}(u,i)$.

One baseline method for rating prediction is based on users' preferences and products' characteristics [11]. Some users have propensity to give higher ratings than others and some products are more likely to receive higher ratings than others. Thus, the optimization formulation to estimate unknown rating $\hat{\mathbf{R}}(u,i)$ is shown below, accounting for both user and product effects:

$$\min_{\mathbf{b},\mathbf{c}} \sum_{(u,i)\in\mathcal{O}} (\mathbf{R}(u,i) - \hat{\mathbf{R}}(u,i))^2 + \lambda(\sum_u \mathbf{b}^2(u) + \sum_i \mathbf{c}^2(i)) \tag{6}$$

where $\hat{\mathbf{R}}(u,i) = \mu + \mathbf{b}(u) + \mathbf{c}(i)$. $\mathbf{b}(u)$ and $\mathbf{c}(i)$ indicate the bias of user u and product i respectively. μ is the average rating over all products and λ controls the regularized part to avoid overfitting.

Another baseline method based on trust networks is the nearest-neighbor algorithm. Let $N(u,i)$ be the set of users who are trusted by user u and have rated product i. Then the unknown rating from u to i can be calculated by:

$$\mathbf{R}(u,i) = \frac{\sum_{v\in N(u,i)} \mathbf{W}(v,u)\mathbf{R}(v,i)}{\sum_{v\in N(u,i)} \mathbf{W}(v,u)} \tag{7}$$

where $\mathbf{W}(v,u)$ is the strength of single trust from u to v.

We want to incorporate the notion of multiple facets into these two baseline methods. The first baseline method assumes that one user has the same bias to all products. However, from the data analysis section, we know that users might have different bias toward different facets. Also the average ratings for different facets are different. Thus $\hat{\mathbf{R}}(u,i)$ can be improved by considering facet differences:

$$\hat{\mathbf{R}}(u,i) = \mathbf{c}(i) + \frac{\sum_k \mathbf{PF}(i,k)(\mu(k) + \mathbf{B}(u,k))}{\sum_k \mathbf{PF}(i,k)} \tag{8}$$

where $\mu(k)$ is average rating for f_k and $\mathbf{B}(u,k)$ is the bias from u toward f_k.

The second baseline method assumes single trust relationships between users. Our tensor representation, i.e., multi-faceted trust relationships, can be easily incorporated into this method as follows:

$$\hat{\mathbf{R}}(u,i) = \frac{\sum_{k=1}^{K}\sum_{v\in N(u,i)} \mathbf{PF}(i,k)\mathcal{A}(v,u,k)\mathbf{R}(v,i)}{\sum_{k=1}^{K}\sum_{v\in N(u,i)} \mathbf{PF}(i,k)\mathcal{A}(v,u,k)} \quad (9)$$

The first model doesn't consider the trust network, while the latter one doesn't consider the bias from products and users. We believe that both factors are useful for rating prediction, thus the rating prediction algorithm to estimate strengths of multi-faceted trust are shown below:

$$\hat{\mathbf{R}}(u,i) = \alpha\Big(\frac{\sum_k \mathbf{PF}(i,k)(\mu(k)+\mathbf{B}(u,k))}{\sum_k \mathbf{PF}(i,k)} + \mathbf{c}(i)\Big) + \quad (10)$$
$$(1-\alpha)\frac{\sum_{k=1}^{K}\sum_{v\in N(u,i)} \mathbf{PF}(i,k)\mathcal{A}(v,u,k)\mathbf{R}(v,i)}{\sum_{k=1}^{K}\sum_{v\in N(u,i)} \mathbf{PF}(i,k)\mathcal{A}(v,u,k)}$$

In this formulation, the rating u giving to i from f_k, $\hat{\mathbf{R}}(u,i)$, is determined by two factors. First, the rating is determined by the bias of u toward f_k and the characteristics of i. This factor is modeled as the first part of our formulation. Secondly, as mentioned above, users in product review sites always seek advice from their trust networks to make decisions. Thus $\hat{\mathbf{R}}(u,i)$ should be influenced by the trust network of u. $\mathcal{A}(v,u,k)$ indicates the strength of trust from u to v in f_k. If u strongly trusts v in f_k, $\hat{\mathbf{R}}(u,i)$ should be similar to $\mathbf{R}(v,i)$. The second part of our formulation is used to model this factor, which captures the influence from multi-faceted trust relationships between users. The parameter α controls the contributions of these two parts. Note that multi-faceted trust relationships can be incorporated into other more complex rating prediction methods, which we leave as our future work.

We define $\mathbf{E}(u,i)$, p and q as following:

$$\mathbf{E}(u,i) \stackrel{\text{def}}{=} \mathbf{R}(u,i) - \hat{\mathbf{R}}(u,i)$$
$$p \stackrel{\text{def}}{=} \sum_{k=1}^{K}\sum_{v\in N(u,i)} \mathbf{PF}(i,k)\mathcal{A}(v,u,k)$$
$$q \stackrel{\text{def}}{=} \sum_{k=1}^{K}\sum_{v\in N(u,i)} \mathbf{PF}(i,k)\mathcal{A}(v,u,k)\mathbf{R}(v,i)$$

To estimate the parameters $\{\mathbf{B},\mathbf{c},\mathcal{A}\}$, we solve the following optimization problem:

$$\min_{\mathbf{B},\mathbf{c},\mathcal{A}} \sum_{(u,i)\in\mathcal{O}} \mathbf{E}^2(u,i) + \lambda(\sum_{u,k}\mathbf{B}^2(u,k) + \sum_i \mathbf{c}^2(i))$$
$$s.t.\;\; \mathcal{A}(v,u,k)\in[0,1] \quad (11)$$

We use projected gradient method to solve Eq. (11). Then the following rules are used to update $\{\mathbf{B}(u,k),\mathbf{c}(i),\mathcal{A}(v,u,k)\}$:

$$\begin{aligned}
\mathbf{B}(u,k) &\leftarrow \mathbf{B}(u,k) - \beta_{\mathbf{B}}\nabla_{\mathbf{B}(u,k)} \quad (12)\\
\mathbf{c}(i) &\leftarrow \mathbf{c}(i) - \beta_{\mathbf{c}}\nabla_{\mathbf{c}(i)}\\
\mathcal{A}(v,u,k) &\leftarrow \begin{cases} 0 & \mathcal{A}(v,u,k) - \beta_{\mathcal{A}}\nabla_{\mathcal{A}(v,u,k)} < 0\\ 1 & \mathcal{A}(v,u,k) - \beta_{\mathcal{A}}\nabla_{\mathcal{A}(v,u,k)} > 1\\ \mathcal{A}(v,u,k) - \beta_{\mathcal{A}}\nabla_{\mathcal{A}(v,u,k)} & else\end{cases}
\end{aligned}$$

where $\beta_{\mathbf{B}}$, $\beta_{\mathbf{c}}$ and $\beta_{\mathcal{A}}$ are the learning step sizes, which are chosen to satisfy *Goldstein Conditions*. $\nabla_{\mathbf{B}(u,k)}$, $\nabla_{\mathbf{c}(i)}$ and $\nabla_{\mathcal{A}(v,u,k)}$ are shown as follows:

$$\begin{aligned}
\nabla_{\mathbf{B}(u,k)} &= -\alpha\mathbf{E}(u,i) + \lambda\frac{\mathbf{B}(u,k)}{\sum_k \mathbf{PF}(i,k)} \quad (13)\\
\nabla_{\mathbf{c}(i)} &= -\alpha\mathbf{E}(u,i) + \lambda\mathbf{c}(i)\\
\nabla_{\mathcal{A}(v,u,k)} &= (\alpha-1)\mathbf{E}(u,i)\frac{\mathbf{PF}(i,k)\mathbf{R}(v,i)p - q\mathbf{PF}(i,k)}{p^2}
\end{aligned}$$

4. APPLYING mTrust

In this section, we apply mTrust to improve some data mining tasks on product review sites: rating prediction, facet-sensitive ranking, and strengthening status theory.

4.1 Rating Prediction

Users in product review sites are likely to refer to product reviews provided by their trust networks. Thus trust have been widely used to improve the performance of rating prediction [16]. Most of these approaches assume single trust relationships. However, our work indicates the existence of multi-faceted trust relationships between users, which can be used to further improve the performance of prediction.

In section 3.2, we have already shown how to model multi-faceted trust relationships to improve nearest-neighbor algorithm. Actually our multi-faceted trust relationships can also be incorporated into other complex rating prediction methods, such as latent factor model [11] as follows:

$$\hat{\mathbf{R}}(u,i) = \alpha\mathbf{p}_u^\top\mathbf{q}_i + (1-\alpha)\frac{\sum_{k=1}^{K}\sum_v \mathbf{PF}(i,k)\mathcal{A}(u,v,k)\mathbf{R}(v,i)}{\sum_{k=1}^{K}\sum_v \mathbf{PF}(i,k)\mathcal{A}(u,v,k)} \quad (14)$$

where $\mathbf{p}_u$ is a user-factors vector associated with user u and $\mathbf{q}_i$ is a product-factors vector with product i. α is also used to control the contributions of two factors in Eq. (14).

4.2 Facet-Sensitive Ranking

Ranking nodes in a network is an important problem, and *HITS* [10] and *PageRank* [21] are two very popular ranking techniques. Most traditional ranking techniques give higher ranks to nodes with better connectivity. However, from the analysis above, people trust others because they have more similar multi-faceted interests, which suggests that ranking algorithms in product review sites should consider both trust networks and people's interests. Multi-faceted trust relationships, which are estimated by taking into account both the link structure and people's multi-faceted interests, can be used for facet-sensitive ranking to improve the performance of ranking.

Let $\mathbf{FR}\in\mathbb{R}^{n\times K}$, each element of which, $\mathbf{FR}(i,k)$, denotes the ranking of u_i in f_k. The k^{th} column of $\mathbf{FR}$, $\mathbf{FR}(:,k)$, represents the rankings of users in f_k. Based on the estimated $\mathcal{A}$, $\mathbf{FR}(:,k)$ can be calculated as following:

$$\mathbf{FR}(:,k) = d\times\mathcal{A}(:,:,k)\times\mathbf{FR}(:,k) + (1-d)\times\frac{\mathbf{e}}{n} \quad (15)$$

where $\mathbf{e}\in\mathbb{R}^n = \{1,1,\dots,1\}$ and the parameter d is used to control the probability that a user would "jump" to some users instead of following the trust relationships.

The overall rankings, $\mathbf{fr}\in\mathbb{R}^n$, can be obtained from the aggregation of the rankings in different facets.

$$\mathbf{fr} = \mathbf{FR}\times\mathbf{r} \quad (16)$$

where $\mathbf{r} \in \mathbb{R}^K$ is a weight vector and $\mathbf{r}(k)$ represents the weight assigned to f_k and associated $\mathbf{FR}(:, k)$.

With the rankings of users in each facet, it is easier for advertisers to differentiate effective users for their products propagation with different facets.

4.3 Strengthening Status Theory

One important social-psychological theory is status theory [13]. In this theory, u_i trusting u_k means that u_i regards u_k as having higher status than herself. However, this theory holds weakly on the subset of links in these networks that are reciprocated(consisting of directed links in both directions between two users) [13]. For example, if u_i trusts u_k and u_k also trusts u_i, as shown in the top subgraph of Figure 7, then it will be difficult for status theory to determine whose status is higher.

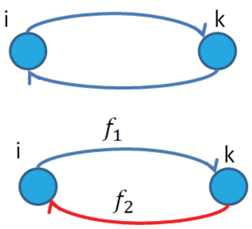


Figure 7: Reciprocity and Status Theory

In the data analysis section, we study reciprocity in trust networks in Epinions and Ciao. The reciprocity is **19.28%** in Epinions and **23.77%** in Ciao. Thus status theory might not hold strongly in both datasets. However, our work indicates that a pair of reciprocal links might be heterogeneous. As shown in the bottom subgraph of Figure 7, u_i trusts u_k in facet f_1 and u_k trusts u_i in facet f_2. According to status theory, u_i has a higher status in f_2 and u_k has a higher status in f_1. In this case, status theory holds for this pair of reciprocal links. Thus our work can be applied to strengthening status theory in networks with reciprocal links.

5. EXPERIMENTS AND FINDINGS

In this section, we describe in detail the findings and different experiments resulting from applying mTrust to the Epinions and Ciao datasets. Two parameters of mTrust, i.e., λ and α, are determined through cross validation.

5.1 Findings about mTrust

For this experiment, all ratings are used as training data to estimate $\mathcal{A}$, which indicates multi-faceted trust relationships between users. We observe the following about $\mathcal{A}$:

- In the studied six facets for both datasets, less than 1% of users trust their friends in all six facets. However, more than 70% of users trust their friends more than one facet. On average, people trust only 35.4% of their trust networks for a specific facet.

- We examine transitive and co-citation trust relationships in both datasets. 22.3% and 17.1% of transitive trust relationships are heterogeneous for Epinions and Ciao, respectively, while 13.1% and 11.7% of co-citation relationships are heterogeneous. Examples of heterogeneous transitive and co-citation trust relationships are shown in Figure 8. For example, a transitive trust relationship, $i \to j \to k$, is heterogeneous. u_i trusts u_j in f_1, however, u_j trusts u_k in f_2; a co-citation trust, $i_1 \to j_2$ and $i_2 \to j_2$, is heterogeneous.

- We study pairs of reciprocal links in the original networks and we find that 23.5% and 24.1% pairs of these links are heterogeneous in the Epinions and Ciao, respectively. As shown in the bottom subgraph of Figure 7, the link $i \to j$ and link $j \to i$ are different types.

- The original network is a big component for both datasets. However, we examine the trust relationships in each facet and there exist many components, most of which are singletons. For example, for the first facet in Epinions, 96.7% of these components are singletons and the biggest component contains 75.5% users.

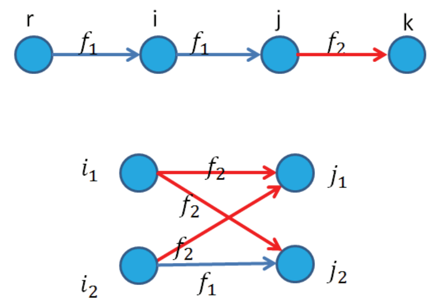


Figure 8: Heterogeneous Transitive and Co-citation Trust Relationships

5.2 Rating Prediction

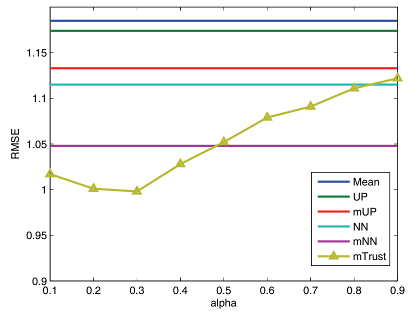
The ratings in both datasets are sorted in *chronological order* and we choose the top 70% of the datasets as training data and the rest as testing data. Then the model parameters are estimated from *past* data and to predict *future* data. The estimated parameters, i.e., $\{\mathbf{B}, \mathbf{c}, \mathcal{A}\}$, can be used to predict unknown ratings. mTrust can be incoporated into other rating prediction methods thus our objective is not to compare rating prediction methods. Instead, we want to verify that considering multi-faceted trust relationships between users allows us to improve the performance of prediction. We use a common metric, Root Mean Squared Error(RMSE), to evaluate prediction accuracy.

$$RMSE(\mathcal{U}) = \sqrt{\frac{\sum_{(u,r)\in\mathcal{U}} (\mathbf{R}(u,i) - \hat{\mathbf{R}}(u,i))^2}{|\mathcal{U}|}} \tag{17}$$

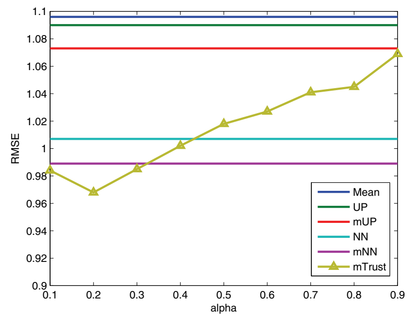
where $|\mathcal{U}|$ is the size of testing data $\mathcal{U}$

Through cross validation, we set $\lambda = 0.05$. The parameter α controls the contributions of two factors of our formulation. We test different values of α to investigate the importance of these two factors in our study datasets. Results are shown in Figure 9. The methods mentioned in the figure are defined as follows:

- *Mean*: the rating of a product is predicted by the mean of known ratings of the product.



(a) Epinions



(b) Ciao

Figure 9: Performance of Rating Prediction in Epinions and Ciao

- *UP*: the rating of a product is predicted by Eq. (6) and this model assumes that one user has the same bias to all products.
- *mUP*: the model is a variant of *UP*, which incorporates the facet differences such as average facet ratings differences and users' facet bias differences.
- *NN*: it refers to the nearest neighbor algorithm, as shown in Eq. (7). *NN* assumes single trust relationships between users.
- *mNN*: the rating of a product is predicted by Eq. (9), which considers multi-faceted trust relationships between users.
- *mTrust*: the rating of a product is predicted by the combination of *mUP* and *mNN*, as shown in Eq (11).

The first observation is that *Mean* and *UP* are very close in performance on both datasets. Most of the time, the majority of users actual ratings are close to the average. As reported in the data analysis section, more than 70% users give a score of 4 or 5. When facet differences are incorporated into *UP*, the performance of *mUP* is apparently improved. This directly supports the existence of facet differences, i.e. average facet rating differences and facet bias differences.

The performance of *NN* is better than that of *mUP* especially on the Ciao dataset. People on product review sites often refer to the reviews from their trust networks. Thus their product ratings can be easily influenced by their trust networks. Ciao has more close-knit trust relationships and people are influenced more by their trust networks. We check the estimated single trust matrix $\mathbf{W}$ and find that the strengths of single trust relationships vary between users, i.e., people trust some people more than others.

The difference between *NN* and *mNN* is that *mNN* assumes multi-faceted trust relationships between users. Comparing the performance of *mNN* with *NN* on both datasets, we can see that *mNN* is always better. The experiments show that mTrust helps capture fine-grained relationships between users for better performance.

For both datasets, *mTrust* obtains the best performance. The algorithm achieves the peak performance with $\alpha = 0.3$ for Epinions dataset and $\alpha = 0.2$ for Ciao dataset. A small α means that a higher weight would be put on *mNN*. This suggests (1) the importance of trust networks in rating prediction, and (2) that an appropriate combination of these two factors is crucial to achieve better performance.

5.3 Facet-Sensitive Ranking

Facet-sensitive ranking, i.e., ranking users using multi-faceted trust relationships, is one important application of mTrust. For instance, advertisers can easily differentiate effective users for product propagation from different facets. Rankings of users in different facets can also be used for the recommendation of helpful reviews that may be buried in a large number of spam reviews [15].

Rankings in facet f_k, $\mathbf{FR}(:,k)$, are calculated by Eq. (15) and the overall rankings, $\mathbf{fr}$, are obtained by Eq. (16). The weight vector $\mathbf{r}$ is set as the probabilities of different facets' presence, calculated according to the number of rated products from corresponding facets. The multi-faceted trust relationships, $\mathcal{A}$, are estimated by all ratings and the damping factor d is set to 0.85. Due to the limitation of space, we only show the results in Epinions since we get similar results in Ciao. Table 5 lists top-5 users for the studied six facets in Epinions.

We observe that *the top ranking people are different for different facets*. In other words, people have different rankings in different facets. The ranking results of Epinions for each facet seem reasonable. For example, "Howard_Creech" is the lead reviewers in "Electronics". His reviews mainly focus on products in "Electronics" and most of his reviews are rated as very helpful. "dlstewart" is the leader in "Home & Garden". "popsrocks" is the leader in "Hotels & Travel" and among the top reviewers in "Restaurants". "Bryan_Carey" is the top 10 people in Epinions. He wrote $3,493$ reviews about products from various facets such as "Restaurants" and "Hotels & Travel". The remaining of the top 5 users are also among the top advisors or top reviewers in their respective facets.

Although "jo.com" is the top 1 user in Epinions, she is not even among the top 5 users in some facets such as "Electronics" and "Computer Hardware". Facet-sensitive ranking for product review effectively take into account both link structure and people's multi-faceted interests.

It is beyond the scope of this paper to probe further causes of high rankings. However, we observe some commonalities among these top users:

- They usually registered at the sites very early. For example, "Howard_Creech" was a member of Epinions since Aug 16,1999 and "yusakugo" registered Epinions on Apr 14, 2000.

Table 5: Top 5 Users in Each Facet in Epinions

Electronics	Home & Garden	Computer Hardware	Hotels & Travel	Restaurants	Kids & Family	overall
Howard_Creech	dlstewart	yusakugo	popsrocks	Bryan_Carey	Freak369	Bryan_Carey
dkozin	Bryan_Carey	Gr8ful	mrkstvns	popsrocks	marytara	Freak369
yusakugo	mountainhigh	lawman67	AliventiAsylum	jo.com	melissasrn	popsrocks
nick1326	Freak369	ptiemann	Bryan_Carey	Bruguru	three_ster	yusakugo
paulphoto	michiman1	paulphoto	jo.com	AliventiAsylum	pippadaisy	marytara

- They contributed many reviews such as "Freak369" wrote $5,921$ reviews and "popsrocks" wrote $2,651$ reviews. More than 90% of their reviews are considered as very helpful. For example, 93% of reviews written by "dkozin" are rated very helpful by other users.
- They have many trustors. For example, "ptiemann" has $2,809$ trustors and "Bryan_Carey" has $1,570$ trustors.

5.4 Strengthening Status Theory in Networks with Reciprocal Links

Status theory is one of important social theories about social networks. It is reported in [13] that the theory of status holds weakly on these networks that are reciprocated. Here we show how mTrust can strengthen status theory in trust networks with reciprocal links.

In status theory, if u_i trusts u_j, it means that u_i regards u_j as having higher status than herself. In the context of product review sites, the facet-sensitive rankings for users can be considered as their statuses in each facet. if u_i trusts u_j in f_k then u_j should have a higher ranking than u_i in f_k, i.e., $\mathbf{FR}(j,k) > \mathbf{FR}(i,k)$.

For this experiment, multi-faced trust relationships, $\mathcal{A}$, are estimated by all ratings. Given the trust relationships in f_k, $\mathcal{A}(:,:,k)$, let $\mathcal{S}$ be the set of all occurrences of form $i \rightarrow j$ in $\mathcal{A}(:,:,k)$. We compute the ratio of trust relationships(RAT), on which status theory doesn't hold as Eq. (18):

$$RAT = \frac{\sum_{(i,j)\in\mathcal{S}} \varphi(\mathbf{FR}(j,k) - \mathbf{FR}(i,k))}{|\mathcal{S}|} \tag{18}$$

where $|\mathcal{S}|$ is the number of trust relationships. $\varphi(x)$ is a function defined as:

$$\varphi(x) = \begin{cases} 0, & if\ x > 0 \\ 1, & if\ x \leq 0 \end{cases} \tag{19}$$

The RAT and Reciprocity(RCT) for trust relationships in each facet and original networks in Epinions and Ciao datasets are reported in Table 6. The reciprocity in trust relationships for each facet is significantly reduced compared to the original trust networks for both datasets. The RAT for each facet is much less than that of the original network. As mentioned above, 23.5% and 24.1% pairs of reciprocal links are heterogeneous in Epinions and Ciao, respectively. With the help of mTrust, status theory becomes stronger in the trust networks with reciprocated links.

Table 6: Status Theory is Strengthened by mTrust

Epinions							
	overall	1	2	3	4	5	6
RAT	0.332	0.228	0.221	0.231	0.207	0.208	0.216
RCT(%)	19.28	14.27	14.23	14.27	14.15	14.16	14.18
Ciao							
	overall	1	2	3	4	5	6
RAT	0.359	0.259	0.263	0.271	0.265	0.265	0.259
RCT(%)	23.77	17.34	17.55	18.37	17.85	17.83	17.49

6. RELATED WORK

In recent years, the notion of trust has attracted more and more attention from computer science communities. [25, 5] investigated the connection between user similarity (such as ratings of movies) and trust. They found a strong and significant correlation between trust and similarity. The more similar two people are, the greater the trust between them is. Guha et al. developed a formal framework of trust propagation schemes [6]. It first separates trust and distrust matrix and then performs operations on them to obtain the transitive trust between two nodes. In [20], the authors propose a method to model and compute the bias or the truthfulness of a user in trust networks. The bias of users are their propensity to trust/distrust other users. They claimed that their model conforms well to other graph ranking algorithms and social theories such as the balance theory. In [14], the authors propose a classification approach to predict if a user trusts another user using features derived from his/her interactions with the latter as well as from the interactions with other uses. In addition, Leskovec et al. study how trust and distrust relations among users in social networks can be predicted using various topological features of a social network [12].

Trust network is an important characteristic of product review sites, where users rely on their trust networks to seek advice and make decisions. In [16], several methods for incorporating trust networks are proposed to improve the performance of collaborative filtering. Matsuo and Yamamoto study and modeled the bidirectional effects between trust relations and product rating [18]. Lu et al. propose a generic framework for incorporating social context information to improve review quality prediction [15]. However, these works assume binary trust relationships between users.

Another direction of related research is the prediction of tie strength. Tie strength prediction differs from trust relationship prediction. The former focuses on modeling the strength of existed links rather than link existence. Based on interaction data, a supervised method is proposed in [9] to distinguish strong ties from weak ties by predicting binary relationship strength between users. Xiang et al. develop a latent variable model to estimate relationship strength from interaction activities and user similarities [24]. In this model, relationship strength is modeled as the hidden effect of user similarities and it also impacts the nature and frequency of online interactions. Au et al. investigated strength of social influence in trust networks [1]. It shows that the strength of trust relation correlates with the similarity among the users. A modified matrix factorization technique is used to estimate strengths of trust relations.

This work assumes single and homogeneous trust relationships between users. However, we show that trust inherently has multiple facets and people may place trust differently to different people.

7. CONCLUSION

In this paper, we study multi-faceted trust between users in the domain of product review sites. A fine-grained approach, mTrust, is proposed to capture multi-faceted trust relationships. We apply mTrust to tasks such as rating prediction, facet-sensitive ranking, and strengthening status theory. Experiments on two real-world datasets show that mTrust effectively capture multi-faceted trust and can be applied to improve the performance of rating prediction, facet-sensitive ranking, and status theory.

There are new research directions to be investigated. First, mTrust does not consider temporal information related to trust networks and product ratings. When a longer time span is studied (e.g., a year), it would be wise to include temporal effects on trust between users. Second, the notion of multi-faceted trust may be applicable for other domains, e.g., following relationships in Twitter. Finally, sophisticated models can be explored to estimate the strengths of multi-faceted trust to advance research and development on trust propagation and trust relationships prediction.

Acknowledgments

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