Equivalence of Hough Curve Detection to Template Matching

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Key Words and Phrases: picture processing, pattern recognition, curve detection, Hough transformation, template matching

CR Categories: 3.63, 6.9

Introduction

Duda and Hart [1] have given an excellent tutorial on the Hough transformation. This transformation can be used to detect mathematical curves in the points of an image by examination of the space of curve parameters. The transformation is quite simple to implement and if the image itself and the parameter space is quantized, an efficient implementation can be achieved, even in sequential software. This communication discusses the hardware detection of straight lines, but the objective is not to espouse an implementation. Instead, we wish to make clear the point that Hough curve detection is template matching and can be better understood by considering it from this viewpoint.

Hardware Implementation Inferred from Software Technique

For a given set of \( n \) image points \( \{(x_i, y_i)\}, \ i = 1, n \) each point \( (x_i, y_i) \) is potentially on any line determined by \( (x_0, y_0) \) and the angle \( \theta \) of a normal to the line from

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Work performed by L.N.K. Corporation, Silver Spring, Maryland, 20904, and sponsored by the 6570th Aerospace Medical Laboratory, Aerospace Medical Division, Air Force Systems Command, U.S. Air Force, Wright-Patterson Air Force Base, Ohio.

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Communications of the ACM

November 1977
Volume 20
Number 11

the origin. If \( \theta \) is quantized to \( T \) levels \( \{\theta_j\}, j = 1, T \) then each point \( (x_i, y_i) \) lies on at most \( T \) lines. If the perpendicular distance from a line to the origin is computed as \( r = f(\theta_j, x_i, y_i) = x_i \cos \theta_j + y_i \sin \theta_j \), then the new coordinate pair \( (\theta_j, r_j) \) represents the line. If \( r \) is quantized to \( R \) levels \( \{r_k\}, k = 1, R \), then the set of all detectable lines is specified and has cardinality \( R \cdot T \).

Quantization in the \((\theta r)\) parameter space makes the use of the Hough transformation practical by simply managing a set of \( R \cdot T \) accumulators. The implementation that follows can be attributed to a suggestion of Rosenfeld [6] who was perhaps the first to publicize the Hough transformation. Sequentially, or in parallel, each image point \( (x_i, y_i) \) is taken and the (at most) \( T \) lines \( \{(\theta_j, r_j)\} j = 1, T \) on which it can lie are computed. Then each of the \( T \) accumulators \( (\theta_j, r_j) \) is incremented. After all points are processed, a simple thresholding of the \( R \cdot T \) accumulators can detect the presence of linearity in the original image domain: if five points make a line, then a threshold of 5 would be appropriate. Merlin and Farber [4] give more details on general curve detection using a parallel computer.

Suppose that a picture is imaged onto a grid (spatial retina) of \( \mathbb{P} = \{x, y\} \) photoelectric cells \( \{(x_i, y_i)\}, i = 1, m \). For each cell \( (x_i, y_i) \) of the spatial retina, the \( T \) possible lines \( \{(\theta_j, r_j)\} j = 1, T \) are determined by \( r_j = x_i \cos \theta_j + y_i \sin \theta_j \). Output lines are connected to cell \( (x_i, y_i) \) and are labeled uniquely \( \{(\theta_j, r_j)\} j = 1, T \) (see Figure 1). In the event some \( r_j \) is outside of the quantization range for \( r \), there will be fewer than \( T \) output lines. Since each output line will carry the same current, the assignment of labels to the lines can be done in any of \( T! \) ways. When output lines have been connected to all cells \( (x_i, y_i) \) in this manner, the inputs to the cells of the Hough retina are easily connected. To cell \( (\theta_k, r_k) \) of the Hough retina are connected all lines labeled \( (\theta_k, r_k) \) emanating from the spatial retina. The Hough retina cells are made of integrating logic, which sums all inputs, compares the sum with a threshold and gives a binary output. A simple "OR" circuit can be used to combine the outputs of all single Hough retina cells into one binary response which would indicate "line detected" or "no line detected." Either analog or digital implementation should be possible.

Hardware Connections Inferred from Inversion of the Software Technique

The previous wiring scheme showed how the response from each spatial cell \( (x_i, y_i) \) is distributed to the appropriate set of \( T \) cells in the accumulator array. This distribution of response from the spatial retina to the Hough retina has been the viewpoint of previously reported research [1–6]. In this section, the scheme is to consider set \( S \) of all spatial cells being integrated by a single accumulator cell \( (\theta_k, r_k) \). \( S \) is in fact the template for line \( (\theta_k, r_k) \). \( S \) is a linear band of cells in the spatial retina and is easily determined to be \( \{S \} = \{(x_i, y_i)\}: x_i \cos \theta_k + y_i \sin \theta_k = r_k \}\) where \( \theta_k \) is used