The States of Dilation in Iris Recognition
(A Preliminary Study)

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Abstract—The impact of pupil dilation on the matching accuracy of iris recognition algorithms has been demonstrated in the biometrics literature. However, the current literature does not model the various states of pupil behavior with respect to the underlying dynamics. Consequently, most existing work on this topic is empirical in nature. Our work uses concepts of transition processes and limiting distributions to describe the relationship between the state of the input iris image and a countable number of enrolled dilation states from an iris recognition standpoint. We also investigate a special case where a closed form expression is obtained that directly relates the various states to overall pupil behavior. Numerical evaluations demonstrate the feasibility of our proposed model where the results of our work can be directly used by iris recognition algorithms to account for pupil dilation artifacts.

I. INTRODUCTION

Iris recognition has made great strides over the past two decades [1], [2]. By taking advantage of the near-infrared spectrum (NIR) to reveal textural details, iris recognition has evolved into a promising biometric technology [3]. Although iris recognition are currently being used for many commercial and government applications, such as access control, there are still limitations to this technology. Recent studies have demonstrated the negative impact of pupil dilation on the matching accuracy of iris recognition systems [4]. Hence, there is a need to explore methods to mitigate the effects of various pupil sizes on matching accuracy.

A. Related Work

Research into the effects of pupil dilation on iris recognition first began with the work by Ma et al. [5] who noted that there were a number of false non-matches due to pupil dilation. Thornton et al. [6] showed that accounting for the level of iris deformation, via Bayesian estimation, leads to an improvement in iris recognition performance. Later, Wei et al. [7] adopted Wyatt’s model [8] to empirically model the effects of deformation as the sum of a linear stretch and a Gaussian deviation term. Motivated by the need to explore the overall effects of dilation on iris recognition, Hollingsworth et al. [4] performed an extensive empirical investigation and concluded that dissimilarities in dilation affect overall matching accuracy.

Recent studies have explored this topic, both empirically and theoretically, from various standpoints. From a deformation perspective, Clark et al. [9] used the principles of biomechanics to model the effects of iris deformation by incorporating the effects of sphincter and dilator muscles. In their work, existence and uniqueness conditions were developed and numerical simulations showed the efficacy of their approach. Later, from a template standpoint, Tomeo-Reyes and Chandran [10] performed bit error analysis to explore the consistency of iris texture information across various degrees of dilation. Their results showed that, when image comparisons are made between the extreme degrees of dilation to the average dilation, bit errors increased by over 10%. Furthermore, from a dilation viewpoint, Gejji et al. [11] revisited the work of [12] to abstractly express the effects of dilation in the visible spectrum and the NIR spectrum. Their work showed that, for the case of steady illumination, Pamplona’s model to represent the typical pupillary response, can be expressed as

\[
\frac{dM}{dt} = -\bar{a}M(t) + f(M(t - \tau)) \quad \text{for } t \geq 0
\]

\[
M(t) = \psi_M(t) \quad \text{for } 0 \leq t \leq \tau
\]

(1)

where \(\psi_M(t)\) is the history function, \(f(M(t - \tau))\) is given by

\[
f(M(t - \tau)) = 5.2 - \bar{\gamma} \ln \left[ \frac{\pi I^*}{4\phi} \right] - 2\bar{\gamma} \ln(D(t - \tau)),
\]

(2)

and \(D\) is expressed as

\[
D = 3 \tanh(M) + 4.9.
\]

(3)

From equation (1), Gejji et al. proposed a methodology to quantify the subject-specific effects of dilation. Testing their methodology on the West Virginia University (WVU) dilation dataset [13] showed that the predicted response closely aligned with the observed response, with less than 2% deviation.

These extreme degrees of dilation include both constriction and dilation cases.
B. Our Motivation and Contribution

Our motivation for this work comes from the literature in [4], [9], [10] and [11] that suggest a need to develop an analytical and universal approach to describe the effects of dilation on iris recognition caused by ambient lighting. The works of [4] and [10] explore this concept from a template perspective; however, this is mainly from an empirical point of view, which is dependent on the technology and data at hand and does not fully consider the physiological effects of the pupil. Additionally, the development of the mathematical models in [9] and [11] illustrate that the iris muscle activity, governed by the sphincter and dilator muscles, determines changes in both pupil size and iris deformation. These changes, described in the works of [4], [10] and [14], indicate that they also affect the associated iris templates. Therefore, our approach views each level of pupil dilation as a “state”, which is inherently associated with an iris template.

The contributions of this work are three-fold. First, we build our theoretical model by considering the empirical formulation of [4] to explore the relationship between the state of the input iris image (hitherto referred to as the acquired state) to the various stored states in an iris recognition database for an average subject. We do this by incorporating the properties of transition processes with the pupillary response, described in [15], [16] and [11], where the result is a transport equation with random initial conditions. Second, we perform theoretical analyses of our transport equation where we consider evolutionary solutions and an approximate solution to our model by considering short temporal duration. Finally, we perform numerical evaluations to show the efficacy of our approach and similarity in behavior of our models with analytical results. Our results advance the prior art because we are theorizing the impact of dilation, which can be used as a feedback mechanism to perform optimal filtering and selection. The rest of this paper is organized as follows: Section 2 presents the analytical development; Section 3 presents the numerical simulations; and Section 4 describes our conclusions and future directions.

II. Analytical Development

\begin{align}
D(t) \quad & D_1 \quad \vdots \quad D_N \\
\downarrow \quad & \quad \downarrow \\
& \vdots \\
& \downarrow \\
& D_t \quad \vdots \\
& \downarrow \\
& \downarrow \\
& D_N
\end{align}

Fig. 1. Abstraction of Hollingsworth’s experimental arrangement that compares the acquired dilation \(D(t)\) to a large number of dilations \(D_1, \ldots, D_N\) from a single subject in an iris database.

Hollingsworth’s empirical arrangement [4], shown in Figure 1, can be viewed as a comparison between the acquired state to the countably large number of enrolled states in an iris database. Hence, Hollingsworth’s arrangement can be modeled as a Markovian process where each state has an associated dilation. Our overall goal is to find the likelihood such that

\[ W(D,t) = P\{D(t) = D\} \] (4)

where \(W(D,t)\) is the probability density function, \(D\) is the dilation that is associated with the enrolled states, and \(D(t)\) is the dilation associated with the acquired state within the dilation domain \(D = (D_{\text{inf}}, D_{\text{sup}}) \subset \mathbb{R}\). We solve for (4) by investigating the equivalent problem

\[ X(M,t) = P\{M(t) = M\} \] (5)

where the probability density function \(X(M,t)\) is valid within the domain \(M = (M_{\text{inf}}, M_{\text{sup}}) \subset \mathbb{R}\). Here, \(M = M(D)\) is the associated value with the enrolled states and \(M(t) = M(D(t))\) is the associated value with the acquired state. Our model formulation for \(X(M,t)\) is done by first making the assumption that for each state \(M_i\) there exists a transition probability \(p^i(M,t|\psi,s)\) such that

\[ p^i(M,t|\psi,s) = P\{M(t) = M|M(s) = \psi\} \] (6)

where \(s < t\). Consequently, we can express (6) as a Taylor series approximation with respect to the time delay \(t - s > 0\) as

\[ p^i(M,t|\psi,s) = \delta(M - \psi) + p^i_1(M,s|\psi,s)(t-s) + O(|t-s|) \] (7)

where \(p^i_1(M,s|\psi,s)\) is defined as

\[ p^i_1(M,s|\psi,s) = \lim_{ds \to 0, ds > 0} \frac{1}{ds} \{p^i(M,s+ds|\psi,s) - \delta(M - \psi)\} \] (8)

and \(\delta\) is the dirac delta function. Next, we make the assumption that \(p^i(M,s|\psi,s)\) and \(p^i_1(M,s|\psi,s)\) are tempered distributions associated with each state. Thus, we describe the changes in these distributions from the transition rate defined by

\[ \lambda_{ij} = \lim_{dt \to 0} \frac{P\{M(t+dt) = M_j|M(t) = M_i\}}{dt}. \] (9)

Additionally, we define \(X^i(D,t)\) as the conditional probability density function expressed as

\[ X^i(M,t) = P\{M(t) = M|M = M_i\}. \] (10)

Using the Chapman-Kolmogorov relation [17], substituting equation (7), and allowing \(dt \to 0\) we obtain the following expression for \(X^i(D,t)\) [15]:
Fig. 2. Illustration of our proposed Markovian process. The development of our model considers the transition between the tempered distributions, $p_i$ and $X^i$, for each state $M_i$ while considering the collection of overall states.

$$\frac{\partial X^i}{\partial t} = \langle p_i^t(M, t|\rho, t), X^i(\rho, t) \rangle + \sum_{i \neq j} (\lambda_{ij}X^j(M, t) - \lambda_{ji}X^i(M, t)).$$

From (11), we note that the first term is defined as

$$\langle p_i^t(M, t|\rho, t), X^i(\rho, t) \rangle = \int_M p_i^t(M, t|\rho, t)X^i(\rho, t)d\rho$$

where $\rho$ represents all of the possible intermediate states. From the conditional probability function $X^i(M, t)$ we express the joint probability for each state by

$$X^i(M, t)\mu_i(t) = P\{M(t) = M \cap M = M_i\}$$

where $\mu_i(t) = P\{M = M_i\}$. The likelihood expression in (13) relates the acquired state $M(t)$ to an enrolled state $M_i$; however, we need to consider all of the possible states. This can be expressed via the joint probability density function $X(M, t)$ given by

$$X(M, t) = \sum_i X^i(M, t)\mu_i(t)$$

where the $\mu_i$’s satisfy the system of ordinary differential equations [16]:

$$\frac{d\mu_i}{dt} = \sum_{i \neq j} (\lambda_{ji}(t)\mu_j(t) - \lambda_{ij}(t)\mu_i(t)).$$

Next, we can make the supposition that approaching the different states of dilation are equally likely and therefore, $\mu_i = \mu_j = \mu$. Hence, from equation (15), our Markov process reduces to a Poisson process where

$$\lambda_{ij} = \lambda_{ji} = \lambda.$$

Noting that the pupil diameter is the driving force of our process, we employ equation (1) and note that $p_i^t(M, t|\psi, t)$ can be expressed as

$$p_i^t(M, t|\psi, t) = -\delta(M - \psi)H(\psi, t, \tau)$$

where

$$H(M, t, \tau) = -\bar{\alpha}M(t) + f(M(t - \tau))$$

and $f(M(t - \tau))$ is defined by equations (2) and (3). Using equation (14) and making the assumption that the initial distribution is normal $N(\mu_M, \sigma_M)$, we achieve the following initial value problem (IVP) to describe $X(M, t)$

$$\frac{\partial X}{\partial t} = -\frac{\partial}{\partial M}\{H(M, t, \tau)X(M, t)\}$$

with the random initial condition

$$X(M, 0) = N(\mu_M, \sigma_M)$$

where $\mu_M$ and $\sigma_M$ respectively represent the mean and variance of the dilation mappings that are associated with $M(D)$. Finally, observing the monotonicity of $M(D)$, we apply the change of variables theorem [18] such that

$$W(D, t) = \left| \frac{dM}{dD} \right| \cdot X(M, t).$$

To find the likelihood $W(D, t)$ (4), we employ the change of variables theorem to both the general solution $X(M, t)$ and the initial condition $X(M, 0)$. We note that, for existence and uniqueness to hold, $H(M, t, \tau)$ needs to be continuous in the domain $M$.

A. Behavioral Evolution of Solutions

For our model, care must be taken to account for the neurological delay $\tau$. Analogous to the method of steps [19], we treat $M(t)$ and $M(t - \tau)$ as independent variables and make the following declarations:

$$X_1 = P(M(t) = x)$$

$$X_2 = P(M(t) = x, M(t - \tau) = \psi_M)$$

$$X_3 = P(M(t) = x, M(t - \tau) = y, M(t - 2\tau) = \psi_M)$$

where $X_2$ and $X_3$ are the joint probability distribution functions corresponding to the incremental delays. Applying Liouville’s theorem [16], we describe the evolutionary response of $X(M, t)$ as

$$\frac{\partial X}{\partial t} + \nabla \cdot (HX) = 0$$
where \( X = [X_1, X_2, X_3]^T \) and \( H \) is defined by equation (18). This evolutionary system, described by (23), represents a hierarchy system of equations to describe the changes in the associated dilation states.

### B. Exploring the Special Case of Short Time Duration

Next, we return to equation (19) and consider the case of short time duration such that \( t \in [0, \tau] \). Hence, our advection equation (19) simplifies to

\[
\frac{\partial X}{\partial t} = - \frac{\partial}{\partial M} \{ H(M, t) X(M, t) \}, \tag{24}
\]

which comprises of the following characteristic equations:

\[
\begin{align*}
\frac{dt}{ds} &= 1, \tag{25} \\
\frac{dM}{ds} &= H(M, s), \tag{26} \\
\frac{dX}{ds} &= - \frac{\partial H}{\partial M} X \tag{27}
\end{align*}
\]

where \( s \) is the parameterized variable. For the first equation (25), we achieve the trivial solution \( s = t \); however, for the second (26), we achieve the following solution as

\[
M_0 = e^{\bar{\alpha} t} M(t) - \int_0^t e^{\bar{\alpha} \zeta} f(\exp(M(\zeta))) d\zeta \tag{28}
\]

where \( f \) is defined by (2). Incorporating the third characteristic equation (27) while considering the initial condition (20), we achieve the final solution for \( X(M, t) \)

\[
X(M, t) = X(M_0, 0) \cdot \exp \left( - \int_0^t \frac{\partial H(M, \eta)}{\partial M} d\eta \right) \tag{29}
\]

where \( H \) and \( M_0 \) are defined by equations (18) and (28), respectively. From our solution (29), we can explore the propagation behavior along the characteristic curves, instead of the evolutionary system (23).

### III. Numerical Simulation

This section presents the numerical simulations of our probabilistic model (19), our evolutionary response (23), and our solution for short time duration (29). In our simulations, we assume our initial condition follows that of a standard Normal distribution \( \mathcal{N}(\mu_D, \sigma_D) \) where \( \sigma_D = 0.1 \) and the mean \( \mu_D = 3.41 \) and \( \mu_D = 7.11 \) for the cases of constriction and dilation, respectively\(^2\). Next, to obtain the initial condition (20), we apply the change of variables theorem [18] such that

\[
X(M, 0) = \left( \frac{dM}{dD} \right)^{-1} \cdot W(D, 0). \tag{30}
\]

\(^2\)The values of \( \mu_D \), for both cases of constriction and dilation were chosen based on the steady state calculations of Pamplona’s model of the pupil light reflex (PLR) described in [11]. The variance \( \sigma_D \) is chosen for purely simulative purposes.

Lastly, after obtaining our solution \( X(M, t) \), we apply (21) to obtain the final solution for \( W(D, t) \). Our numerical comparison involved employing two numerical techniques. We simulated the response of (19) via Monte Carlo methods where 10,000 simulations were performed using 1,000 bins between dilation values of \( D_{inf} = 1.9 \) and \( D_{sup} = 7.9 \) [12]. Additionally, for improved fitting, we weighed each simulation using a Gaussian kernel with standard deviation bandwidth of 0.01. To simulate our evolutionary system (23), we used classical upwind finite differencing. Our direct solution for short time duration (29) was implemented by numerically examining the characteristic curve (26) for \( t \in [0, \tau] \) where we chose \( \tau = 250 \) milliseconds [11].

Fig. 3. Simulation of responses from equations (19), (23), and (29) that illustrate the likelihood of observation associated with increased dilation. Here, the initial history (31) is considered with \( \mu_D = 3.41 \) and \( \sigma_D = 0.1 \).

Fig. 4. Simulation of responses from equations (19), (23), and (29) that illustrate the likelihood of observation associated with increased constriction. Here, the initial history (31) is considered with \( \mu_D = 7.11 \) and \( \sigma_D = 0.1 \).

Figures 3 and 4 highlight the likelihood of observation associated with dilative and constrictive responses of the pupil, respectively. For both cases we see a consistent trend between our model (19), the evolutionary equation (23), and our solution (29). Furthermore, we note that our exact solution closely aligns with the proposed model where we see the direct connection between the pupillary response and the impact of selection via the characteristic curves.
IV. DISCUSSION

We theorized the empirical arrangement of [4] to describe the states of dilation from an iris recognition standpoint. We do this by describing the likelihood of selection between the acquired state to any of the stored states in the database that is described as an advection equation with an initial random variable. From this arrangement, we were able to obtain two alternative representations describing this observation as both a system of evolutionary equations as well as an exact solution for small time duration. Each representation supports the works of [4], 20, and 14], because we were able to theorize how distinctions in dilation affect iris recognition accuracy. Moreover, our work is an improvement on the prior art because we were able to provide an abstract foundation to previous empirical observations. Numerical simulations showed the efficacy of our proposed model because we were able to show the consistent relationship between pupil dilation and the likelihood of observation. Additionally, our exact solution for small time showed the direct connection between pupil motion and this probabilistic effect, which can be modified for understanding subject-specific effects.

The results of our work have several applications that can benefit the iris recognition community. Our model can be applied as a predictive methodology for both testing and correcting the effects of dilation on iris recognition. It is noted that some commercial algorithms account for pupil dilation; however, it might not be a closed-loop system. Our model can provide a feedback mechanism to these processes in order to ensure that the optimum dilation is selected for better iris recognition. Consequently, our work could serve to further understand the temporal stability effects of iris recognition systems. From this perspective, our model can provide insights into the impact that additional anatomical changes, such as aging and other ocular disorders, have on iris recognition accuracy. In their report, NIST researchers promoted the need for modeling the effects of known covariates [14]. Our model addresses this need because we were able to provide a generalized foundation of previous empirical results by incorporating the analytical description of the pupillary light reflex [11].

Although our model is an advancement of the prior art and the numerical comparisons of our model yield promising results, there are several areas of future work that need to be explored. First, our model examines the states of dilation from a general perspective. Therefore, one potential area of exploration is understanding the states of dilation from specific perspectives like aging. Another area of exploration is understanding the stability effects of our model to investigate both the controllability and observability. Lastly, there is a need for testing our model using actual data to further connect the developed theory to experimental investigations from both general and definitive viewpoints.

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REFERENCES