Section 2.3

2. Determine whether $f$ is a function from $\mathbb{Z}$ to $\mathbb{R}$ if
   
   (a) $f(n) = \pm n$.
   
   This is not a function because the rule is not well-defined. We do not know whether $f(n) = n$ or $f(n) = -n$. For a function, it cannot be both at the same time.

   (b) $f(n) = \sqrt{n^2 + 1}$.
   
   This is a function. For all integers $n$, $\sqrt{n^2 + 1}$ is a well-defined real number.

   (c) $f(n) = \frac{1}{n^2 - 4}$.
   
   This is not a function with domain $\mathbb{Z}$, since for $n = 2$ and $n = -2$ the value of $f(n)$ is not defined by the given rule. In other words, $f(2)$ and $f(-2)$ are not specified since division by 0 makes no sense.

5. See textbook.

15. Determine whether the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if

   (a) $f(m, n) = m + n$.
   
   Onto.

   (b) $f(m, n) = m^2 + n^2$.
   
   Not onto.

   (c) $f(m, n) = m$.
   
   Onto.

   (d) $f(m, n) = |n|$.
   
   Not onto.

   (e) $f(m, n) = m - n$.
   
   Onto.

19. Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

   (a) $f(x) = 2x + 1$.
   
   Yes.

   (b) $f(x) = x^2 + 1$.
   
   No.

   (c) $f(x) = x^3$.
   
   Yes.

   (d) $\frac{x^2 + 1}{x^2 + 2}$
   
   No.

38. Let $f$ be the function from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = x^2$. Find

   (a) $f^{-1}(\{1\})$.
   
   $\{1, -1\}$

   (b) $f^{-1}(\{x|0 < x < 1\})$.
   
   $\{x| -1 < x < 0 \lor 0 < x < 1\}$

   (c) $f^{-1}(\{x|x > 4\})$.
   
   $\{x|x > 2 \lor x < -2\}$
73. For each of these partial functions, determine its domain, codomain, domain of definition, and the set of values for which it is undefined. Also, determine whether is is a total function.

(a). \( f : \mathbb{Z} \to \mathbb{R}, \ f(n) = \frac{1}{n}, \)

Domain is \( \mathbb{Z} \); codomain is \( \mathbb{R} \); domain of definition is the set of nonzero integers; the set of values for which \( f \) is undefined is \( \{0\} \); not a total function.

(c). \( f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Q}, \ f(m, n) = \frac{m}{n}. \)

Domain is \( \mathbb{Z} \times \mathbb{Z} \); codomain is \( \mathbb{Q} \); domain of definition is \( \mathbb{Z} \times (\mathbb{Z} - \{0\}) \); set of values for which \( f \) is undefined is \( \{0\} \); not a total function.

Not from the book

1. Consider the sets \( A = \{0, 1, 2, 3, 4\} \) and \( B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \). Let \( f \) be a function from \( A \) to \( B \), whose graph is \( G_f = \{(0, 0), (1, 2), (2, 4), (3, 6), (4, 8)\} \)

(a) Give the range of \( f \).

\( \{0, 2, 4, 6, 8\} \)

(b) If the inverse, \( f^{-1} \), exists, give it as a set of ordered pairs. If it does not exist, say why not.

The inverse does not exist because the function \( f \) is not onto.

(c) Let \( g(x) = \left\lfloor \frac{x}{2} \right\rfloor \) be a function from \( B \) to \( A \)

i. Give the domain of the composition function \( g \circ f \).

\( \{0, 1, 2, 3, 4\} \)

ii. Give the codomain of \( g \circ f \).

\( \{0, 1, 2, 3, 4\} \)

iii. Give the graph of \( g \circ f \) as a set of ordered pairs.

\( \{(0,0), (1,1), (2,2), (3,3), (4,4)\} \)