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- **Hash table**: a totally different data structure achieving expected constant searching time.
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Hash table $T$ contains an array storing $m$ pointers.
Hash Function

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Two different kinds of Hashing: chaining and open address.
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Hash: Chaining

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Finding a good hash function is the key (needs very complicated proof by number theory and probabilistic analysis, and not covered by this course).
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- In practice, we often use $h(x) = x \mod p$, where $p$ is a prime number.
An example: suppose the stream of numbers are 10, 6, 11, 20, 21, 13, 25, 71, 15, 12, ......  
Hash function: $h(x) = x \mod 7$. 

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>

- 0
- 1
- 2
- 3
- 4
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- 6
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<tr>
<td>10</td>
<td>6</td>
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\begin{itemize}
  \item ![Hashing Example](example_hashing.png)
\end{itemize}
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```
0  21
1
2
3  10
4  11
5
6  6  20
```
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```
0  21
1  71 15
2
3  10
4  11 25
5
6  6 20 13
```
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![Hash Chaining Diagram](image_url)
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- Hash function \( h(x, i) \), where \( i \) counts the number of trials from 0 to \( m-1 \). Two common ways:
  
  **Linear Probing:**
  
  \[
  h(x, i) = h_1(x) + i \mod m
  \]

  **Double Hashing:**
  
  \[
  h(x, i) = h_1(x) + i \times h_2(x) \mod m
  \]

  where \( h_2(x) \) and \( m \) should be relatively prime.
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A good hash function could yield constant searching time (again, needs very complicated proof by number theory and probabilistic analysis, and not covered by this course).
Hash: Open Addressing

- An example: suppose the stream of numbers are 11, 32, 53, ....
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- \begin{align*}
0 & \quad 0 \\
1 & \quad 1 \\
2 & \quad 2 \\
3 & \quad 3 \\
4 & \quad 11 \\
5 & \quad 5 \\
6 & \quad 6
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- An example: suppose the stream of numbers are 11, 32, 53, ......
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```
0  
1  
2  
3  
4  11
5  
6  32
```
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Insertion And Deletion

Remember that hash table is dynamic data structure (similar to AVL-Tree).

- When inserting $n > m$ elements, double the size $m$ to $2m$, and re-hash all the existing elements.
- When deleting too many elements such that $n \leq m/4$, shrink $m$ to $m/2$, and re-hash all the existing elements.

The amortized complexity for $n$ insertions/deletions is $\theta(n)$.

Note: for open addressing hash table, keep a "flag" when deleting an element for future search.
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