Outline

1. Context and Background Material
2. Fuzzy Ant System
3. Bee System
4. Q&A and Discussion

Transportation Modeling: An Artificial Life Approach*

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Combinatorial Optimization Problems (COPs)

- Finite set $S$ of feasible solutions
- Objective function $f : S \rightarrow \mathbb{R}$
- Optimal solution: $s \in S$ that minimizes or maximizes $f(s)$

Context

- Using Artificial Life methods
  - Fuzzy Ant System
  - Bee System
- To solve combinatorial optimization problems
Some Transportation-Related COPs

- Traveling Salesman Problem
- Vehicle Routing Problem
- Highway Alignment Optimization
- Transit Line Schedule Synchronization
Why is TSP Hard?

• Previous example:
  • 3 choices for first node
  • 2 choices for second node
  • 1 choice for third node
  • $3 \times 2 \times 1 = 6$ possible solutions
• In general, $O(n!)$ possible solutions
• $n!$ grows very quickly: $20! > 2 \times 10^{18}$

Example: Traveling Salesman Problem
**Hardness**

- Class P: Can **solve** in polynomial time
- Class NP: Can **verify** solution in polynomial time
- NP-hard: At least as hard as any NP problem
- Strategy: Seek *near*-optimal solutions

**Big-Oh and Polynomial Time**

![Graph showing Big-Oh and Polynomial Time](image)
Swarm Intelligence

- Inspired by behavior of social insects
- Individuals follow simple rules
- Communication between individuals
  - Ants leave pheromone trail
  - Bees dance to advertise food sources
- Intelligent behavior emerges from these interactions

Strategies Inspired by Nature

- Genetic Algorithms
- Simulated Annealing
- Neural Nets
- Swarm Intelligence
Artificial Ants and the TSP

An artificial ant:
1. Has memory
2. Makes non-deterministic choices
3. Communicates via pheromones

Ant System

- Proposed by Colorni, et al in 1991
- Inspired by behavior of ants in nature
- Used to solve Traveling Salesman Problem
- Primary inspiration for Bee System and Fuzzy Ant System
Ant System Example

Ant 1 Start

Ant System Example
Artificial Pheromone

- After each iteration, pheromone on edge \((i, j)\) is updated:

\[
t_{ij} = (\text{evaporation} \times t_{ij}) + (\text{ant updates})
\]
Artificial Pheromone

• After each iteration, pheromone on edge \((i, j)\) is updated:

\[ t_{ij} = (\rho \times t_{ij}) + \left( \sum_{k=1}^{m} (I_{kij} \times \Delta_k) \right) \]

• Where:
  • \(\rho\) is the evaporation rate
  • \(k\) represents the \(k^{th}\) ant
  • \(m\) is the number of ants
  • \(I_{kij}\) indicates (1 or 0) whether ant \(k\) visited \((i, j)\)
  • \(\Delta_k\) is the change in pheromone due to the \(k^{th}\) ant

Artificial Pheromone

• After each iteration, pheromone on edge \((i, j)\) is updated:

\[ t_{ij} = (\rho \times t_{ij}) + (\text{ant updates}) \]

• Where:
  • \(\rho\) is the evaporation rate
Artificial Pheromone

- After each iteration, pheromone on edge \((i, j)\) is updated:

\[ t_{ij} = (\rho \times t_{ij}) + \left( \sum_{k=1}^{m} (l_{kij} \times \frac{Q}{L_k}) \right) \]

- Where:
  - \(\rho\) is the evaporation rate
  - \(k\) represents the \(k^{th}\) ant
  - \(m\) is the number of ants
  - \(l_{kij}\) indicates (1 or 0) whether ant \(k\) visited \((i, j)\)
  - \(Q\) is an arbitrary constant
  - \(L_k\) is the length of the tour taken by the \(k^{th}\) ant

Ant System Example

- Set \(\rho = 0.5\)
- Set \(Q = 100\)
- Set \(L_1 = 25\)
- Set \(L_2 = 30\)

\[ \Delta_1 = \frac{Q}{L_1} = \frac{100}{25} = 4 \]

\[ \Delta_2 = \frac{Q}{L_2} = \frac{100}{30} = 3.3 \]
Ant System Example

Ant 1 Start, Iteration 2

5; 4.5
9; 3.8
9; 7.8
4; 4.5

7; 7.8

5; 3.8

Ant System Example

5; (0.5*1 + 4) = 4.5
9; (0.5*1 + 3.3) = 3.8
7; (0.5*1 + 3.3 + 4) = 7.8

9; 7.8

5; 3.8

4; 4.5
Probability of Node Selection

• An ant at node $i$ travels to node $j$ with probability:

$$Prob(j) = \frac{(t_{ij})^\alpha \times (visibility\ of\ (i,j))}{normalization\ factor}$$

• Where:
  • $t_{ij}$ is the quantity of pheromone on $(i, j)$
  • $\alpha$ is a tuning parameter
Probability of Node Selection

• An ant at node $i$ travels to node $j$ with probability:

$$Prob(j) = \frac{(t_{ij})^\alpha \times (n_{ij})^\beta}{\sum_{k \in U} (t_{ik})^\alpha (n_{ik})^\beta}$$

• Where:
  • $t_{ij}$ is the quantity of pheromone on $(i, j)$
  • $\alpha$ is a tuning parameter
  • $n_{ij}$ is the inverse of the distance from $i$ to $j$
  • $\beta$ is a tuning parameter
  • $U$ is the set of unvisited nodes

Probability of Node Selection

• An ant at node $i$ travels to node $j$ with probability:

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Schedule Synchronization

• Given:
  • A transit system with transfer stations
  • Line headways
  • Stop time at stations
  • Travel time between stations
  • Number of transferring passengers
• Goal: Minimize waiting time by optimizing terminal departure times
Fuzzy Set Theory

- Set membership can be partial and can overlap:

![Graph showing membership degrees for Short, Medium, and Tall categories with a scale from 0 to 1 on the y-axis and Height on the x-axis.]

- Each element has membership degree
- E.g., Bob is Short with truth degree 0.4
- Truth degree ≠ Probability

Schedule Synchronization

- Given:
  - A transit system with transfer stations
  - Line headways
  - Stop time at stations
  - Travel time between stations
  - **Number of transferring passengers**

- Goal: Minimize waiting time by optimizing terminal departure times
Fuzzy Ant System (FAS)

- Modifies Ant System
- Replaces exact values with fuzzy sets
  - Distance
  - Pheromone intensity
- Replaces transition probability with Fuzzy Logic
FAS for Schedule Synchronization

Model passengers transferring from line $i$ to line $j$ at time $u$ as:

\[ P_{iju} \]

FAS for Traveling Salesman

- Fuzzy transition function:
  
  If distance is **SHORT** and trail intensity is **STRONG**
  Then utility is **VERY HIGH**

- Probability calculated for each target node based on fuzzy utility
- Otherwise the same as Ant System
“Ride” line, computing arrival, departure, and wait times.

Possible Terminal Departure Times
FAS for Schedule Synchronization

Ant 1

Ant 2
FAS for Schedule Synchronization

- On next iteration, ants pick next node using fuzzy rules that consider:
  - Estimated wait time
  - Pheromone levels
  - Node selection based on rules such as

    If wait time is **SMALL** and trail intensity is **STRONG** Then utility is **VERY HIGH**
Behavior of Bees in Nature

1. Find Food Source
2. Return Nectar to Hive
3. Choose one of the following:
   a. Return to food source alone
   b. Perform “waggle dance”
   c. Abandon food source

Bee System

• Inspired by foraging behavior of bees in nature
• Similar to ideas found in the Ant System
• Used to solve Traveling Salesman Problem
Bee System Example

Stage 1, Iteration 1

S = 2 (two nodes visited per stage)
Bee System Example

Stage 1, Iteration 1

S = 2 (two nodes visited per stage)
Bee System: Time Hierarchy

1. $s$ nodes visited during a stage
2. $\left\lceil \frac{|N|-1}{s} \right\rceil$ stages during an iteration
3. $M$ iterations

Bee System Example

Stage 2, Iteration 1

S = 2 (two nodes visited per stage)
Probability of Choosing a Node

Probability that bee $k$ at node $i$ during stage $u + 1$ and iteration $z$ chooses node $j$:

$$p_{ij}^k (u + 1, z) = \frac{\exp(-a d_{ij} \frac{z}{\text{popularity}})}{\text{normalization factor}}$$

Where:

- $a$ is a tuning parameter
- $d_{ij}$ is the distance from node $i$ to node $j$
- $z$ is the iteration index
Probability of Choosing a Node

Probability that bee $k$ at node $i$ during stage $u + 1$ and iteration $z$ chooses node $j$:

$$p_{kij}(u + 1, z) = \frac{\exp(-ad_{ij})}{\sum_{r=\max(z-b,1)}^{z} n_{ij}(r)}$$

Where:
- $a$ is a tuning parameter
- $d_{ij}$ is the distance from node $i$ to node $j$
- $z$ is the iteration index
- $b$ is the memory length
- $n_{ij}(r)$ counts the bees that visited link $(i, j)$ during iteration $r$

Probability of Retaining Tour

Probability that bee $k$ does not abandon their partial tour after stage $u$ during iteration $z$:

$$p_k(u + 1, z) = \exp\left(-\frac{L_k(u, z) - \min(L_r(u, z))}{u \times z}\right)$$

Where:
- $L_k(u, z)$ is the length of bee $k$’s tour during stage $u$ and iteration $z$
**Waggle Dancing**

- Bees usually advertise tours they retain
- Communication is via waggle dancing
- Bees that abandon their tour watch dancing bees to select a new tour
- The probability of following a dancer:
  - Increases with shorter tour lengths
  - Increases with more bees advertising the same tour

**Benchmark Results**

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Optimal Value</th>
<th>Best Bee System Value</th>
<th>%-Difference</th>
<th>Run Time (seconds)</th>
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</thead>
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<tr>
<td>Eil51</td>
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</tr>
</tbody>
</table>
Uncertainty: Algorithm

- Source: Behavior of Algorithm
- Mitigation: N/A
- Leveraging controlled uncertainty

Time Complexity
Uncertainty: Environment

- Sources:
  - Trains:
    - Arrival / travel times
    - System malfunctions
    - Number of transfer passengers
  - Other:
    - Cultural expectations
    - Geography
- Mitigation?

Uncertainty: State Space

- Source: State space size
- Solution space: Is solution optimal?
- Input space: Is algorithm validated?
- Mitigation:
  - Non-deterministic search
  - Benchmark Comparisons
Limitations

- Tuning parameter guidance
- Validation
- Time complexity
- Justification for probability functions
- Justification for complexity
- Many types of uncertainty not considered

Uncertainty: Environment

- Sources:
  - Trains:
    - Arrival / travel times
    - System malfunctions
    - **Number of transfer passengers**
  - Other:
    - Cultural expectations
    - Geography
  - **Mitigation: Fuzzy Logic**

- Buses and trucks:
  - Weather
  - Road conditions
  - Traffic
  - Driver behavior
  - Policy Changes
Research Challenges

• How do we validate these systems?
• Can we improve the time complexity?
• How can we handle other types of uncertainty?
• Might different mathematical models improve results or running time?
• How might these systems be applied to dynamically adaptive systems?

References

Questions and Discussion

References


