

# Computer Science 830: Design and Theory of Algorithms

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## Homework #1 Due September 17<sup>th</sup> 2009, 10:20am

Each full problems should be solved on a separate sheet of paper to facilitate grading. Limit the solution of each sub-problem to at most one side of a sheet of paper. The numbered problems all come from Introduction to Algorithms, by Cormen, *et al.* Please do not wait until the last minute to look at the problems.

1. In the directory `~cse830/Public/hw1/` on `arctic.cse.msu.edu`, there is a collection of six short C language programs and six unidentified executable files, all of whose time complexities are a function of  $n$ . Each executable accepts an integer value  $n$  as its command line argument.

- Using the Unix `time` command, run each executable for multiple values of  $n$ , and graph the results. Run "`man time`" for more information on how the command works. Make sure the graphs are clear enough to justify your answers.
- Remember to choose your values of  $n$  carefully in order to get a good idea of what these curves really look like. For example, you might start with: `time exe-A 1`
- Look at the source code of the C language programs. Match up each executable with its source program.

2. Give a proof or counter-example for each of the following statements:

- a. if  $f(n) = O(F(n))$  and  $g(n) = O(G(n))$ , then  $f(n)/g(n) = O(F(n)/G(n))$
- b.  $f(n) = O(g(n))$  implies that  $2^{f(n)} = O(2^{g(n)})$
- c. For all functions  $f(n)$  and  $g(n)$ , either  $f(n) = O(g(n))$  or  $g(n) = O(f(n))$ .

3. For each of the following pairs of functions  $f(n)$  and  $g(n)$ , determine whether  $f(n) = O(g(n))$ ,  $g(n) = O(f(n))$ , both, or neither.

- a.  $f(n) = (n^2 - n)/2$ ,  $g(n) = 6n$
- b.  $f(n) = n + 2 \sqrt{n}$ ,  $g(n) = n^2$
- c.  $f(n) = n \log n$ ,  $g(n) = n \sqrt{n} / 2$
- d.  $f(n) = \sqrt{n}$ ,  $g(n) = n + \log n$
- e.  $f(n) = 2(\log n)^2$ ,  $g(n) = \log n + 1$
- f.  $f(n) = 4n \log n + n$ ,  $g(n) = (n^2 - n) / 2$

4. For each of the following function pairs  $f(n)$  and  $g(n)$ , give a minimal positive constant  $c$  such that  $f(n) < c g(n)$  for all  $n \geq 1$ .

a.  $f(n) = n^2 + n + 1$ ,  $g(n) = 2n^3$

b.  $f(n) = n \sqrt{n} + n^2$ ,  $g(n) = n^2$

c.  $f(n) = n^2 - n + 1$ ,  $g(n) = n^2/2$

5. Order of magnitude calculations: These questions will explore your ability to estimate numbers within a couple of orders of magnitude. Don't worry about the *correct* answer, I'm interested in how you make your assumptions and follow through on them.

- Estimate the total amount of money in an armored car completely filled with \$20 bills.
- Estimate how many cubic miles of water flow out of the mouth of the Mississippi River each day.
- Estimate the number of Lego blocks it would take to build a copy of the statue of liberty that is the same size as the original.

**Do not** look up any supplemental facts, but feel free to ask me any questions you have about the exercise. Describe all assumptions you made in arriving at your answer.

6. Exercises 4.1-5 and 4.2-3