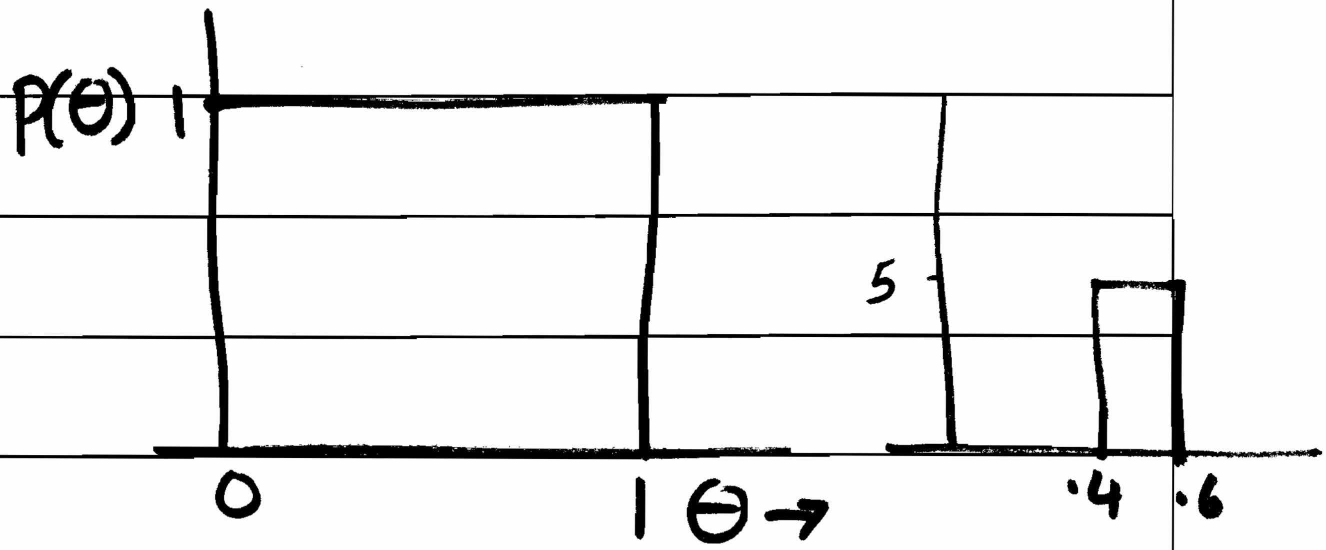
Θ = PROB. OF HEADS $0 \leq \Theta \leq 1$



UNIFORM PRIOR

n_H = # HEADS IN n TOSSES OF COIN

$\hat{\Theta}_{MLE} = n_H/n$

$$p(n_H|\theta) = \binom{n}{n_H} \theta^{n_H} (1-\theta)^{n-n_H}$$

$$p(\theta|n_H) = \frac{p(n_H|\theta)\,p(\theta)}{\int_0^1 p(n_H|\theta)\,p(\theta)\,d\theta}$$

$$\int_0^1 p(n_H|\theta)\,p(\theta)\,d\theta = \binom{n}{n_H} \int_0^1 \theta^{n_H} (1-\theta)^{n-n_H}\, d\theta$$

$$= \binom{n}{n_H} \frac{\Gamma(n_H+1)\,\Gamma(n-n_H+1)}{\Gamma(n+2)}.$$

$$= \frac{1}{n+1}$$

$$p(\theta|n_H) = (n+1)\binom{n}{n_H} \theta^{n_H} (1-\theta)^{n-n_H}.$$

FOR SQUARED-ERROR LOSS FN.

$$\hat{\theta}_{BAYES} = \int_0^1 \theta\, p(\theta | n_H)\, d\theta$$

$$= \frac{n_H + 1}{n+2} \qquad \hat{\theta}_{MLE} = \frac{n_H}{n}$$

① SUPPOSE $n = 0$,

$\hat{\theta}_{MLE}$ UNDEFINED

$\hat{\theta}_{BAYES} = \frac{1}{2}$!!

② $n = 1, n_H = 0$

$\hat{\theta}_{MLE} = 0$

$\hat{\theta}_{BAYES} = \frac{1}{3}$

③ n LARGE

$$\frac{n_H + 1}{n+2} \to \frac{n_H}{n}$$