BAYES CLASSIFIER FOR MULTIVARIATE GAUSSIAN DENSITY WITH UNKNOWN COVARIANCE MATRIX

(D. G. KEEHN, "A NOTE ON LEARNING FOR GAUSSIAN PROPERTIES", IEEE TRANS. INFO. TH., JAN 1965)

- $f(\underline{x} \mid P) = N(0, \Sigma^{-1}) \qquad P = \Sigma^{-1}$

$$= (2\pi)^{-K/2} \, |P|^{1/2} \exp\left\{-\frac{1}{2} \underline{x}^t P \underline{x}\right\}$$

K = NO. OF FEATURES

- REPRODUCING PRIOR FOR MATRIX P IS A <u>WISHART DENSITY</u>

- $f(p_{11}, p_{12} \cdots, p_{KK}) = C_{K,\nu_0} \left| \Phi_0 \frac{\nu_0}{2} \right|^{\frac{(\nu_0 - 1)}{2}} \cdot |P|^{\frac{(\nu_0 - K - 2)}{2}} \cdot$

$$\exp\left[-\frac{1}{2} tr \, \nu_0 \Phi_0 P\right]$$

$$C_{K,\nu_0} = 1 \Big/ \left\{ \pi^{\frac{K(K-1)}{4}} \cdot \prod_{d=1}^{K} \Gamma\left(\frac{\nu_0 - d}{2}\right) \right\}$$

- ϕ_0 IS A +VE DEFINITE MATRIX
- ν_0 IS A REAL NUMBER, $\nu_0 > k$

- APOSTERIORI DENSITY OF P IS ALSO A WISHART DENSITY WITH PARAMETERS ν_n AND ϕ_n

$$f(p_{11}, p_{12} \cdots, p_{kk} \mid x_1, x_2 \cdots, x_n) \propto$$

$$|P|^{\frac{(\nu_0+n-k-2)}{2}} \cdot \operatorname{Exp}\left[-\frac{1}{2} \operatorname{tr} (\nu_0+n) P \frac{\nu_0 \phi_0 + n \langle \underline{x}\, \underline{x}^t \rangle}{\nu_0+n}\right.$$

$$\nu_n = \nu_0 + n$$

$$\phi_n = \frac{\nu_0 \phi_0 + n \langle x\, x^t \rangle}{\nu_0 + n}$$

$$\langle x\, x^t \rangle = \frac{1}{n} \sum_{i=1}^{n} \underline{x}_i\, \underline{x}_i^t$$

ϕ_n : WEIGHTED AVERAGE OF ... $\equiv$ M.L.E. OF Σ^{-1}

$$f(P) = W(\Phi_0, \nu_0)$$

PRIOR DENSITY

$K \times K$ matrix

Scaler

$$f(P \mid X) = W(\Phi_n, \nu_n)$$

POSTERIORI DENSITY

Φ_n = LINEAR COMB. OF Φ_0 & SAMPLE COVARIANCE MATRIX

$$\nu_n = \nu_0 + n$$

$$f(\underline{x} \mid \underline{x}_1, \underline{x}_2, \dots, \underline{x}_n) \leftarrow f(\underline{x} \mid \Sigma^{-1}) \sim N(0, \Sigma^{-1})$$

$$= (2\pi)^{-k/2} \frac{\Gamma\left(\frac{\nu_n}{2}\right)}{\Gamma\left(\frac{\nu_n - k}{2}\right)} \frac{1}{\left(\frac{\nu_n}{2}\right)^{k/2}} |\phi_n|^{1/2}$$

$$\left(1 + \frac{1}{\nu_n} \operatorname{tr} \underline{x}\,\underline{x}^t \phi_n^{-1}\right)^{-\frac{\nu_n}{2}}$$

- $f(x \mid X)$ IS NO LONGER GAUSSIAN.

HOWEVER, IT DOES APPROACH THE TRUE DENSITY AS $n \to \infty$.

$$\lim_{n \to \infty} \log_e f(\underline{x} \mid X) = \log_e (2\pi)^{-k/2} - \frac{1}{2} \log_e |\Sigma| - \frac{1}{2}\left(\underline{x}^t \Sigma^{-1} \underline{x}\right)$$