NEYMAN – PEARSON DECISION RULE

- CONSIDER A 2-CLASS HYPOTHESIS TESTING PROBLEM
- H_1: X BELONGS TO ω_1
- H_2: X BELONGS TO ω_2
- p = PRIOR PROBABILITY THAT X BELONGS TO ω_1 $= P(\omega_1)$
- $p(x|\omega_1)$, $p(x|\omega_2)$: LIKELIHOOD FUNCTIONS
- DIVIDE THE PATTERN SPACE Ω_X INTO TWO DISJOINT REGIONS, Ω_1 AND Ω_2

  IF $X \in \Omega_1 \rightarrow$ ACCEPT H_1

  IF $X \in \Omega_2 \rightarrow$ ACCEPT H_2
- DECISION FN. $\delta(x)$ s.t. $\delta(x) = \omega_1$ IF $x \in \Omega_1$ & $\delta(x) = \omega_2$ IF $x \in \Omega_2$
- α = PROB. OF ERROR OF FIRST KIND

$$= \int_{\Omega_2} p(x|\omega_1)\,dx \quad \left\{ \begin{array}{l} \text{CONDITIONAL PROB. THAT} \\ \delta(x) = \omega_2 \text{ GIVEN } H_1 \text{ IS TRUE} \end{array} \right.$$

- $$\beta = \int_{\Omega_1} p(x|\omega_2)\,dx \quad \left\{ \begin{array}{l} \text{CONDITIONAL PROB. THAT} \\ \delta(x) = \omega_1 \text{ GIVEN } H_2 \text{ IS TRUE} \end{array} \right.$$

  = PROB. OF ERROR OF SECOND KIND
- $(1-\alpha)$ AND $(1-\beta)$ ARE PROBABILITIES OF A CORRECT DECISION GIVEN H_1 & H_2, RESPECTIVELY.

- LET C_1 = COST OF AN ERROR OF THE FIRST KIND

  C_2 = COST OF AN ERROR OF THE SECOND KIND

- RISK = $c_1 \alpha p + c_2 \beta (1-p)$

$$= c_1 p \int_{\Omega_2} p(x|\omega_1)\,dx + c_2 (1-p) \int_{\Omega_1} p(x|\omega_2)\,dx$$

- BAYES CLASSIFIER WILL DIVIDE Ω_x INTO OPTIMAL DECISION REGIONS IN THE SENSE THAT THE RISK IS MINIMIZED.

NEYMAN- PEARSON CLASSIFICATION

- PRIOR PROBABILITY p AND THE COSTS C_1 AND C_2 ARE NOT AVAILABLE.

- WE ASSUME THAT THE PROB. OF ERROR OF THE FIRST KIND, α IS GIVEN.

- NEYMAN- PEARSON CLASSIFIER WILL MINIMIZE β FOR THE FIXED α.

- THE FIXED α IS CALLED THE <u>LEVEL OF THE TEST</u> AND $(1-\beta)$ IS CALLED THE <u>POWER OF THE TEST</u>

THEOREM

THE NEYMAN-PEARSON CLASSIFIER THAT MINIMIZES β FOR A GIVEN VALUE OF α IS A LIKELIHOOD RATIO TEST WITH THE THRESHOLD T DETERMINED BY THE EQUATION

$$\alpha = \int_{\Omega_2} p(x|\omega_1)\,dx = \int_T^{\infty} h_1(\Lambda)\,d\Lambda = \alpha$$

$h_1(\Lambda) = h(\Lambda|\omega_1)$ = CONDITIONAL PROB. OF $\Lambda(x)$ GIVEN THAT x BELONGS TO ω_1

$$\Lambda(x) = \frac{p(x|\omega_2)}{p(x|\omega_1)}$$

- THE POWER OF THE TEST IS

$$1-\beta = \int_{\Omega_2} p(x|\omega_2)\,dx = \int_T^{\infty} h_2(\Lambda)\,d\Lambda$$

$$h_2(\Lambda) = h(\Lambda|\omega_2)$$