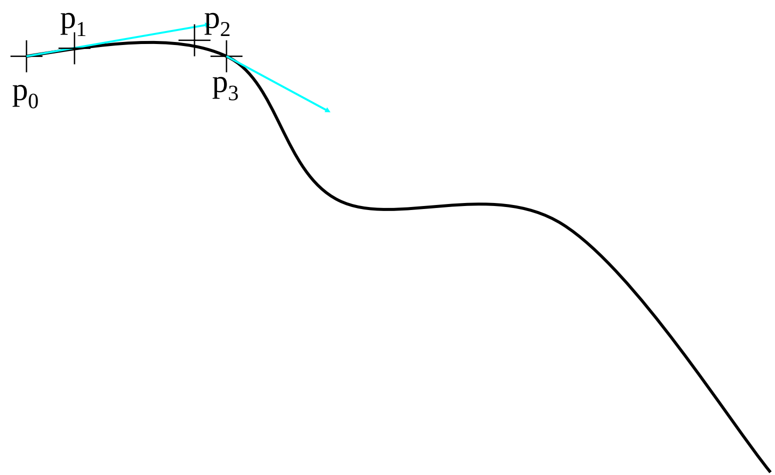
Curves

Curves II

- Blending Functions
- Bernstein Polynomials
- Bézier Surface Patches

A Bézier Curve



Bézier Curve Geometry Matrix

The matrix we computed for the Bézier Curve is called a *Geometry Matrix*

$$M_B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

How a Geometry Matrix is used

$$M_B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

or…

$$c = M_B p$$

$$u = \begin{bmatrix} 1 \\ u \\ u^2 \\ u^3 \end{bmatrix}$$

$$p(u) = \begin{bmatrix} 1 & u & u^2 & u^3 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

$$p(u) = u^T c$$

$$p(u) = u^T M_B p$$

Blending Polynomials

$$M_B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

$$p(u) = \underline{u^T M_B}\, p$$

$$u^T M_B = \begin{bmatrix} 1 & u & u^2 & u^3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 1 - 3u + 3u^2 - u^3 \\ 3u - 6u^2 + 3u^3 \\ 3u^2 - 3u^3 \\ u^3 \end{bmatrix}^T$$

So…

$$p(u) = u^T M_B p$$

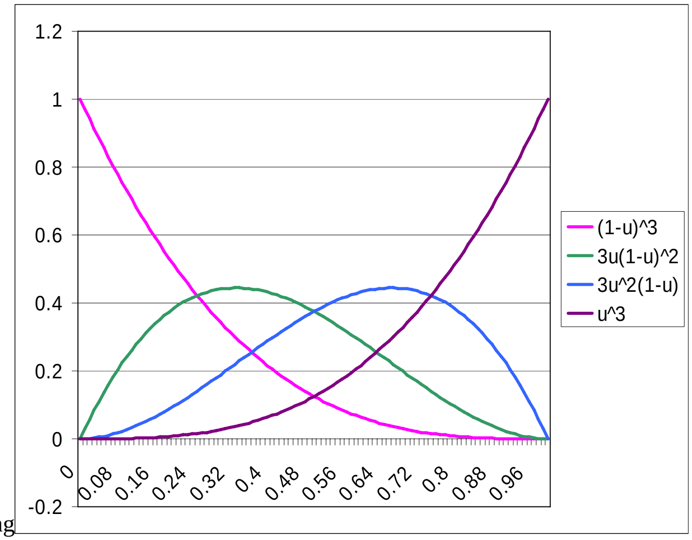
$$u^T M_B = \begin{bmatrix} 1 & u & u^2 & u^3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 1-3u+3u^2-u^3 \\ 3u-6u^2+3u^3 \\ 3u^2-3u^3 \\ u^3 \end{bmatrix}^T$$

b(u)

$$p(u) = \begin{bmatrix} 1-3u+3u^2-u^3 \\ 3u-6u^2+3u^3 \\ 3u^2-3u^3 \\ u^3 \end{bmatrix}^T \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

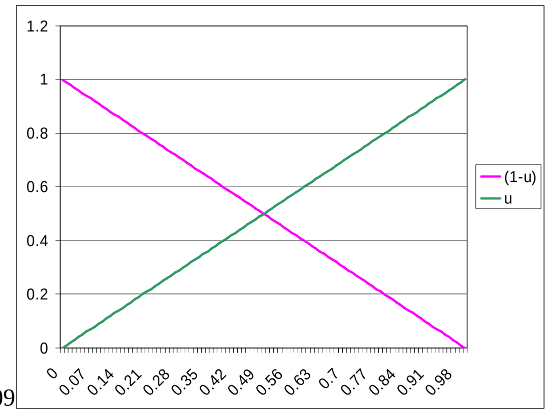
These are called "blending polynomials"

Bézier Blending Polynomials

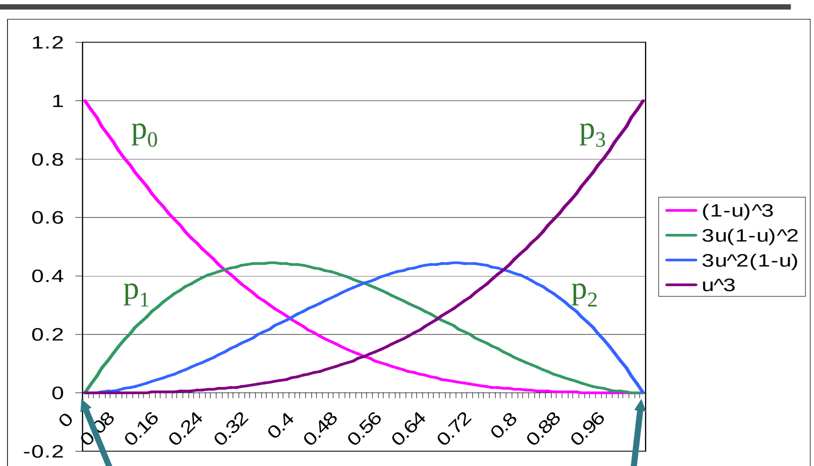


Remember Linear Interp.?

$$p(u) = \begin{bmatrix}(1-u)\\ u\end{bmatrix}^T \begin{bmatrix}p_0\\ p_3\end{bmatrix} = \left[(1-u)p_0 + up_3\right]$$



Bézier Blending Polynomials



Beginning of the curve

End of the curve

Bernstein Polynomials

$$b(u) = \begin{bmatrix} 1 - 3u + 3u^2 - u^3 \\ 3u - 6u^2 + 3u^3 \\ 3u^2 - 3u^3 \\ u^3 \end{bmatrix} = \begin{bmatrix} (1-u)^3 \\ 3u(1-u)^2 \\ 3u^2(1-u) \\ u^3 \end{bmatrix}$$

Bernstein Polynomials:

$$b_{kd}(u) = \frac{d!}{k!(d-k)!} u^k (1-u)^{d-k}$$

d is “degree”, 3 in our case for a cubic.

k is the polynomial number 0-3

Blending func. & our curve…

$$b(u) = \begin{bmatrix} (1-u)^3 \\ 3u(1-u)^2 \\ 3u^2(1-u) \\ u^3 \end{bmatrix}$$

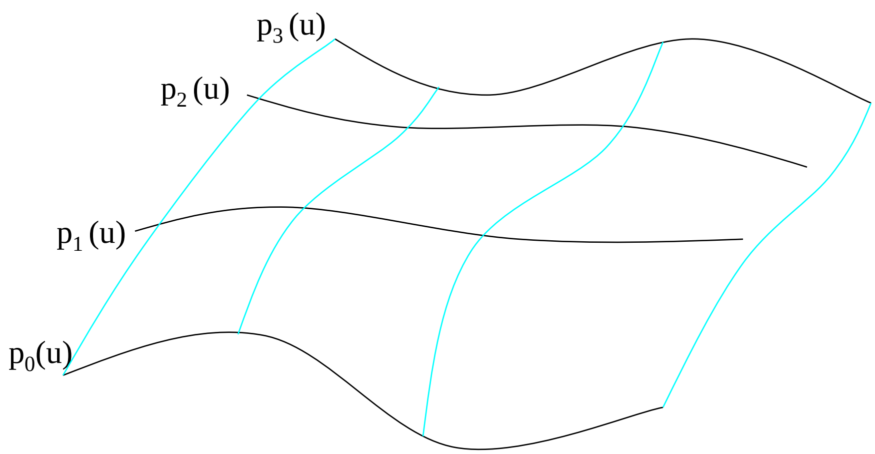
$$p(u) = \sum_{k=0}^{3} b_k(u)\, p_k$$

Extended to a surface

Suppose the controls points were points on a curve?

- Specify each control point for a curve with a Bézier curve
  - We need 4 sets of 4 points = 16 points.

Example



Bézier Surface Patches

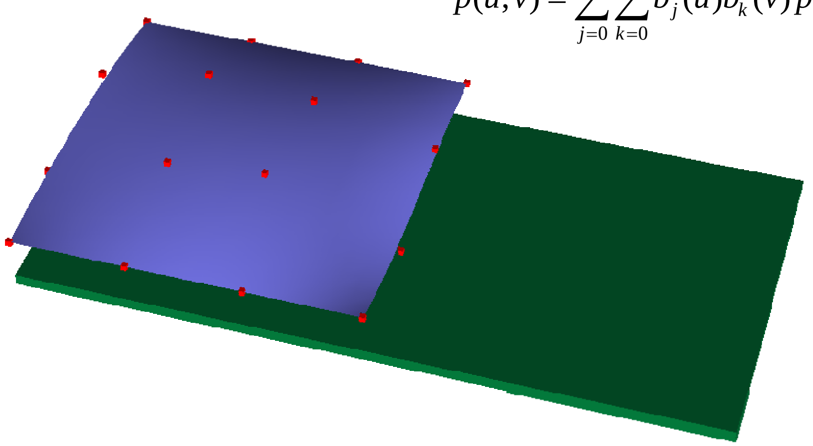
$$b(u) = \begin{bmatrix} (1-u)^3 \\ 3u(1-u)^2 \\ 3u^2(1-u) \\ u^3 \end{bmatrix}$$

Curve: $p(u) = \sum_{k=0}^{3} b_k(u)\, p_k$

Extended to 2D: $p(u,v) = \sum_{j=0}^{3} \sum_{k=0}^{3} b_j(u) b_k(v)\, p_{j,k}$

Example

$$p(u,v) = \sum_{j=0}^{3} \sum_{k=0}^{3} b_j(u) b_k(v)\, p_{j,k}$$



Questions

Continuity?

Which points are on the surface?