Problem 1 (15pt) Hedge Algorithm

In class, we discussed the Hedge algorithm, which learns positive weights to combine the predictions from multiple experts/classifiers. In this problem, you are asked to extend the Hedge algorithm such that the combination weights can be both positive and negative. You need to submit your algorithm and the justification for your algorithm. You will earn optional 10 points if you can prove the mistake/regret bound of your algorithm.

Problem 2 (15pt) Perceptron Algorithm

In the class, we showed that the mistake bound $M_T$ for Perceptron satisfies the following inequality

$$ M_T \leq \frac{1}{2\eta} |w^*|^2 + \sum_{t=1}^T \ell(y_t, x_t^T w^*) + \frac{\eta}{2} R^2 M_T $$

where $w^*$ is the best classifier. In this problem, you are asked to show that with appropriate choice of $\eta$, we have the following inequality for the mistake bound $M_T$

$$ M_T \leq \frac{1}{2} |w^*|^2 R^2 + \sum_{t=1}^T \ell(y_t, x_t^T w^*) + \sqrt{\frac{|w^*|^2 R^2 + 4 \sum_{t=1}^T \ell(y_t, x_t^T w^*)}} $$

You need to give the value of $\eta$ that leads to the above bound.